



Sharing orders or vehicles? Multi-carrier robust service network design problem

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ABSTRACT

The substantial increase in order volume has resulted in a highly competitive freight market, which requires horizontal collaboration among carriers to guarantee cost-effective and timely delivery in transportation. In this case, the collaborative methods implemented by the carriers obviously impact the freight capacities. Hence, to our best knowledge, this paper investigates the collaborative service network design problem under the frameworks of order sharing and vehicle sharing for the first time. Meanwhile, in response to fluctuations in customer demand, we develop a novel two-stage robust optimization model based on the budget uncertainty set for each collaborative framework. Given the NP-hard nature of solving these formulas, an innovative smart-downscale-scenario column-and-constraint generation algorithm is designed to solve the proposed models efficiently, which embeds a linear classifier that identifies scenarios beneficial for convergence. A comparative analysis with the column-and-constraint generation algorithm shows the advantages of our proposed method, which achieves a 25% improvement in solution speed with only a minimal loss in accuracy. In addition, a qualitative analysis of the solution structures is conducted in various collaborative frameworks to demonstrate their flexibility in managing uncertain demand. Based on generated small-scale instances, the results suggest that when the unit outsourcing cost is high, the order sharing framework is more effective for scenarios characterized by low demand volatility; conversely, the vehicle sharing framework is deemed more appropriate for contexts with high demand volatility. Finally, we explore the grand coalition stability through the cooperative game core allocation method.

1. Introduction

In the practical execution of parcel transportation, whether the freight is transported over long distances between cities or between urban transfer centers (see [Liu et al. \(2023\)](#), [Fontaine et al. \(2021\)](#)), in order to enhance both the efficiency and cost-effectiveness of transportation, carriers may need to consolidate packages from many shippers onto a single loading truck to deliver. The consolidation-based freight transportation, including less-than-truckload (LTL) and small package or parcel delivery, plays a critical role in improving the profitability of logistics companies. According to the report¹, the market size for LTL services in China attained 1,228.1 billion

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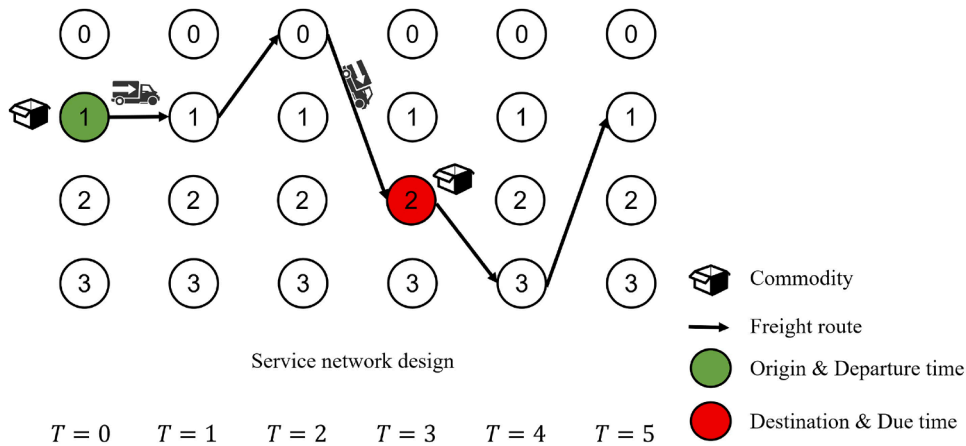


Fig. 1. An instance of the service network.

CNY in 2023, with forecasts indicating sustained expansion within the industry. Meanwhile, expansion in the market size also meant a notable increase in operational freight volume, thereby intensifying pressure and competition within the transportation markets. This trend poses several challenges for consolidation-based carriers, including reduced profit margins and increased expectations for service quality (Verdonck et al., 2013; Crainic and Rei, 2025). To address these challenges, carriers are gradually sharing resources to engage in horizontal collaboration. For example, Changtong Logistics, a logistics company in Henan, China, together with Shandong Jiayi, Anhui Dazhongyuan, and other logistics carriers, set up the Central China Logistics Alliance to realize efficient joint freight. This collaboration ultimately contributes to reductions in transportation costs and improvements in service quality.

In general terms, the resolution of tactical planning problems related to consolidation-based freight transportation systems has long been supported by the operations research methodology, *service network design*. Decisions regarding the service network design problem for consolidation-based carriers involve developing a scheduled table to coordinate truck paths in both the space and time dimensions, aiming to efficiently fulfill the estimated demands at the minimum possible cost (refer to Fig. 1). Despite the numerous advantages associated with horizontal collaboration, there is limited research to discuss collaborative implementation within the context of tactical planning (Gückel et al., 2025). That is, most existing studies assume that individual carriers utilize homogeneous vehicles to carry out goods delivery (Andersen et al., 2009a; Teypez et al., 2010; Boland et al., 2017). To tackle the challenges posed by the exponential growth in order volumes and to derive managerial insights for executing collaboration among logistics companies in freight transportation, in this paper, we focus on the *collaborative service network design problem* (CSNDP).

Similarly to the service network established by a single carrier, the CSNDP also requires that carriers determine when and where the service begins, how often the service runs, and how commodity flow is allocated across these services. Additionally, for CSNDP, it is essential to explicitly identify the participating carrier responsible for providing these services, a factor that is fundamentally influenced by the collaborative framework. In this case, the different collaborative frameworks significantly influence the service network structure, which directly determines both economic efficiency and service quality. Therefore, it is imperative to investigate the variations in scheduling in different collaborative frameworks and evaluate their effectiveness in achieving cost reductions. In this paper, we study the CSNDP in the context of two common collaborative frameworks: order sharing and vehicle sharing, respectively.

Order sharing is a widely studied collaborative model between carriers (such as Handoko and Lau, 2016; Kovtun et al., 2019; Zhang et al., 2022). A relevant illustration is Alibaba's logistics platform, Cainiao Network, which has been developed to consolidate the processing of orders from numerous small and medium-sized carriers, thereby facilitating the centralized deployment of transportation vehicles for delivery operations. In this collaborative model, the resources of multiple carriers are integrated to collectively establish a service network to fulfill customer orders from all participating carriers. Specifically, in a non-cooperative framework, orders can only be fulfilled by the carrier that holds them. In contrast, the order sharing model allows for the fulfillment of individual orders through the collaborative transportation efforts of multiple carriers. Meanwhile, the coalition collectively delivers all orders by determining the most cost-efficient transportation solution across the participating carriers. This collaborative approach capitalizes on the advantages of coalitional integration, which contributes to a reduction in transportation costs. Nonetheless, it is crucial to acknowledge that this collaborative model poses significant challenges in execution compared to other models, primarily due to concerns regarding the customer information privacy (Gansterer and Richard, 2020).

Vehicle sharing is another collaborative model. For example, DPD, a member of the European Federation of Logistics, leases vehicles from its partner company GLS in specific regions to ensure the efficient completion of orders that cannot be fulfilled independently, particularly during peak demand periods or when capacity is limited. Each carrier within the coalition is only responsible for fulfilling its own orders. When certain coalition members experience insufficient capacity while others have idle vehicles, these underutilized vehicles are often loaned to those members lacking adequate capacity, thereby enhancing overall profitability. It is important to distinguish this model from order sharing. In the context of order sharing, freight is transported by the carrier deemed most cost-effective by the coalition. Conversely, under the vehicle sharing model, carriers utilize their own or leased vehicles from

other carriers to meet their own customer demands without exchanging orders. This process eliminates the need for sharing customer information. This collaborative approach is referred to as vehicle sharing (Sahand et al., 2024; Wang et al., 2024). In such instances, additional profit is obtained when the borrowed vehicle successfully fulfills an order that would remain inadequately serviced in a non-collaborative framework.

In summary, in this paper, we study the CSNDP within the two aforementioned collaborative frameworks, aiming to evaluate the influence of different collaborative models on scheduling solutions that determine the economic and operational performance of the service network. In essence, the focus is directed towards the tactical-level decisions undertaken by carriers concerning the configuration of their service networks. However, evaluating the performance of the two collaborative frameworks solely under deterministic settings does not adequately encompass the complexities intrinsic to the real-world problem. This limitation arises from the fact that the rapid development of e-commerce has resulted not only in a significant surge in demand volume but also in the facilitation of a more convenient online shopping way for consumers. That is, customers are able to easily adjust their demand volume with minimal effort. Consequently, fluctuating customer demands engender a considerable challenge for collaborative carriers in terms of freight transportation. Neglecting demand uncertainty during the planning phase may result in severe negative consequences (Lium et al., 2009; Bai et al., 2014; Wang and Qi, 2020). Hence, it is essential to take into account for demand uncertainty within the CSNDP and discuss how different collaborative frameworks mitigate the risks associated with uncertain demand. More importantly, one may provide valuable managerial insights for carriers regarding effective collaboration strategies across different uncertain degrees. To this end, we develop a two-stage robust optimization (TSRO) model for CSNDP in two collaborative models based on a budget uncertainty set (Bertsimas and Sim, 2004).

The main contributions of this paper are concluded as follows: (i) To our best knowledge, this paper represents a pioneering investigation into the CSNDP within the frameworks of order sharing and vehicle sharing. In response to the fluctuations in demand, a novel two-stage robust optimization model has been developed for each collaborative framework. (ii) Given the NP-hard nature of the two-stage robust SND model, there is a pressing need to propose an efficient solution methodology. Consequently, a smart-downscale-scenario column-and-constraint generation (SDS-CCG) algorithm is designed to address the proposed models, which is numerically shown to be computationally efficient and to yield high-quality solutions. (iii) By analyzing solutions under varying degrees of demand uncertainty, we provide valuable managerial insights for logistics companies concerning their practical implementation. The solution structure based on small-scale instances is intuitively illustrated. The results suggest that when unit outsourcing cost is high, an order sharing framework is more beneficial when demand volatility is low, while a vehicle sharing framework is more advantageous in contexts characterized by high demand volatility. Furthermore, this paper employs the core allocation method from cooperative game theory to assess the coalition stability.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature, providing a foundation for understanding the context and significance of this paper. Meanwhile, this section also demonstrates the research gap. In Section 3, we introduce the TSRO model for CSNDP within the two collaborative frameworks, detailing their formulation and key assumptions. Section 4 introduces the reformulation and provides a comprehensive description of the process of the SDS-CCG algorithm. In section 5, we conduct numerical experiments to validate the effectiveness of the model and algorithm, offering insights into their practical implications. Section 6 concludes the paper and presents several promising directions for future research.

2. Literature review

Within certain freight contexts, to achieve economies of scale, carriers need to coordinate truck routes in space and time to consolidate freight demands from different customers. For example, less-than-truckload freight transport and small package/parcel transport. This form of planning can typically be implemented by solving a service network design problem (see Jarrah et al. (2009), Zhu et al. (2014), Boland et al. (2017)), which is often seen as a medium-term planning level for the company. Typically formulated as a capacitated, fixed-charge, multi-commodity network design problem (refer to Crainic and Rousseau (1986), Ghamlouche et al. (2003, 2004), Yaghini et al. (2015)), the service network is responsible for planning what services should be provided when, where, and with what frequency in order to determine commodity flow, the objective of which is to deliver commodities to customers by the deadline at the lowest possible cost. Due to the significant value and extensive application for practical logistics operations, it has been studied extensively in the literature (Crainic, 2000; Crainic et al., 2014; Hewitt, 2022; Liu et al., 2023).

In the logistics sector, there is an increasing emphasis on enhancing the efficiency of asset utilization. Consequently, it becomes essential to incorporate asset management into the planning process of service network design. Pedersen et al. (2009) was the first to incorporate design-balanced (asset-balanced) constraints into service network design problems. These constraints require that the quantity of assets entering and exiting each node must remain equal, resulting in a circular structure for assets throughout the entire planning horizon. By introducing such constraints, logistics service providers can comprehensively evaluate asset utilization across multiple cycles. Andersen et al. (2009b) further explored multi-fleet collaboration in SND and discussed the potential of cooperative services in enhancing resource utilization. Teypaz et al. (2010) and Andersen et al. (2011) designed decomposition schemes and branch-and-price algorithms to solve the large-scale service network design problem with asset balancing, respectively. Based on this, Crainic et al. (2018) further considered the resource acquisition and made the model usable for strategic planning. Obviously, the collaborative design service network involves managing the assets of multiple carriers, necessitating the incorporation of an asset balancing equation.

The above-mentioned research has mainly addressed problems under deterministic settings. In order to render the problem more relevant to real-world operations while avoiding the negative impact of demand fluctuations, Lium et al. (2009) studied the service network design problem by using a scenario-based stochastic optimization method. They found that stochastic solutions increase

scheduling flexibility and robustness through consolidation and path sharing. Bai et al. (2014) extended the study of Lium et al. (2009) by introducing the re-routing strategy, decreasing the conservatism of the stochastic solutions. Wang et al. (2018) qualitatively analyzed the values of deterministic solutions for freight transportation planners under stochastic environments. They demonstrated the possibility of upgrading deterministic solutions to stochastic solutions. While the aforementioned studies hedge the risk of uncertain demand to some extent, they cannot avoid the large negative impact of the worst-case scenario. Hence, based on the budget uncertainty set, Wang and Qi (2020) proposed the robust service network design problem. They designed a column-and-constraint generation algorithm to solve the proposed two-stage robust optimization model. Subsequently, to overcome the conservatism of the robust solution, Xiang et al. (2022) applied an almost robust optimization method (see Baron et al. (2019)). This method allows the nonsatisfaction of partial demand volumes by setting a risk threshold, which makes solutions less expensive. These studies highlight the importance of considering demand uncertainty at the beginning of the planning horizon.

Note that most studies assume that transportation is undertaken by a single carrier using homogeneous vehicles. In practical applications, homogeneous vehicle fleets encounter difficulties in flexibly meeting diverse demand requirements owing to inherent physical characteristics, including capacity and speed, which consequently result in elevated costs. Wang et al. (2019) studied the service network design problem considering a heterogeneous fleet. They proposed a hybrid algorithm including pricing and cutting techniques and demonstrated the advantages of using heterogeneous fleets, which resulted in a reduction in the total transportation cost and fewer empty trips. Meanwhile, Wang and Qi (2019) also addressed a service network design problem considering multiple types of services under demand uncertainty. Subsequent research by Fontaine et al. (2021) presented the economic benefits when carriers design their own service network under multimodal conditions. Specifically, they studied a service network design problem for two-tier multimodal city logistics. Both line-based (e.g., regular rail and tram lines) and non-line (trucks) vehicles provide transportation services at the first tier. Small vehicles or eco-friendly electric vans deliver goods to meet customer demand at the second tier. However, for the sake of discussion, these studies assume that all heterogeneous assets are managed by a single carrier. Indeed, when confronted with a substantial number of orders, an individual carrier typically does not possess adequate resources to ensure the delivery of all orders. In this case, it is necessary for carriers to engage in horizontal collaboration by sharing certain resources.

Although extensive research suggests that collaboration among multiple carriers can offer benefits such as enhanced profitability and reduced environmental pollution, these results have mainly been found and discussed within the setup of vehicle routing problems (see, Guajardo and Rönnqvist (2016), Gansterer and Richard (2018, 2020)). For instance, Mancini et al. (2021) and Zhang et al. (2022) considered the exchange of customer orders among collaborative partners and investigated the collaborative vehicle routing problem within an order sharing framework. Sahand et al. (2024) introduced a collaborative vehicle utilization model, referred to as the vehicle sharing framework. Within this framework, when transport capacity is insufficient, collaborating partners have the option to lease one another's underutilized vehicles to accommodate surplus freight. Both collaborative frameworks offer considerable benefits to collaborative carriers, including substantial cost savings and improved asset utilization. This motivates us to examine the service network design problem under two collaborative frameworks.

There is also little research addressing the collaborative service network design problem within the existing literature. Taherkhani et al. (2022) developed a service network design formula for a freight system integrating multiple stakeholders. Through a central *Intelligent Decision Support Platform* acting as a coordinator, the integration and optimal allocation of multi-carrier resources are achieved. This formula comprehensively considers practical factors such as shipper categories, demand classifications, and bundles of service offers. These considerations have driven the realization of more efficient consolidation and higher profits. Recently, considering that the service areas covered by a single carrier are limited, Zhang et al. (2024) studied a CSNDP. In their settings, some commodities exceed the delivery range of a single carrier, thereby requiring multiple carriers to collaborate to complete the delivery, regarded as a cross-regional transfer model. Specifically, delivering a single commodity may require crossing multiple carriers' service areas, making it difficult for a single carrier to complete the task. Compared to non-collaborative models, collaborative carriers undertake the transportation of these commodities within their respective service areas until delivery, thereby earning additional profits. There has been no research conducted on the CSNDP within the frameworks of order sharing and vehicle sharing. To address this gap, in this paper, we study the CSNDP within two broader collaborative frameworks: order sharing and vehicle sharing. Both collaboration models are mainly based on resource integration and optimization to achieve more flexible and cost-effective delivery, rather than necessitated by the need to complete goods beyond the scope of operations.

In summary, significant research has modeled and solved the service network design problem from different perspectives, providing theoretical support and valuable managerial insights. Nevertheless, as the freight market becomes increasingly competitive, carriers gradually tend to improve their overall profitability through developing strategic partnerships. To address these challenges, in this paper, we study CSNDP and aim to answer two crucial questions: Firstly, **how do the different collaborative models affect the performance of the service network**, and secondly, **how can the best collaborative model be selected in different uncertainty demand scenarios?** Hence, we study CSNDP under two distinct collaborative frameworks: order sharing and vehicle sharing. We further considered asset management and demand uncertainty, formulating the problem into a two-stage robust optimization model. Given the NP-hard nature of the problem, a generalized SDS-CCG algorithm is proposed to solve the proposed models. To highlight the contributions of this paper, we summarize the most relevant works in Table 1.

3. CSNDP formulation

In this section, we describe CSNDP in detail within two collaborative frameworks, introducing the parameters and decision variables necessary for their formulation. Finally, a qualitative discussion is provided, analyzing the differences and connections between the two models.

Table 1

The works are most relevant to our CSNDP.

Paper	Carrier		Uncertainty			Algorithm		Collaborative Model
	single	collaboration	Determ	SO	RO	exact	inexact	
(Ghamlouche et al., 2003)	✓		✓				✓	
(Ghamlouche et al., 2004)	✓		✓				✓	
(Pedersen et al., 2009)	✓		✓				✓	
(Andersen et al., 2009a)	✓		✓					
(Lium et al., 2009)	✓			✓				
(Andersen et al., 2011)	✓		✓			✓		
(Bai et al., 2014)	✓			✓				
(Zhu et al., 2014)	✓		✓				✓	
(Crainic et al., 2014)	✓		✓				✓	
(Boland et al., 2017)	✓		✓				✓	
(Wang and Qi, 2019)	✓				✓	✓		
(Wang et al., 2019)	✓		✓				✓	
(Wang and Qi, 2020)	✓				✓	✓		
(Fontaine et al., 2021)	✓		✓			✓		
(Xiang et al., 2022)	✓					✓		
(Zhang et al., 2024)		✓			✓	✓		Cross-regional transfer
This paper		✓			✓		✓	Sharing orders/vehicles

3.1. Order sharing

Order sharing is a widely studied model of collaboration in the literature (see [Handoko and Lau \(2016\)](#), [Kovtun et al. \(2019\)](#)). Within the confines of this theoretical framework, the fundamental assumptions can be outlined as follows (see [Zhang et al. \(2022\)](#), [Sahand et al. \(2024\)](#)): (1) Multiple collaborative carriers; (2) Each carrier has customer orders to serve and its own vehicle fleets; (3) Participating collaborative carriers can fulfill each other's orders; and (4) the carrier's service possesses the capacity to ensure coverage for all nodes. It is assumed, based on the above, that there exists a group of carriers $h \in \mathcal{H}$ who cooperate in addressing a CSNDP under the framework of order sharing, aiming to design a scheduling vehicle scheme that serves customer demands at the lowest possible cost. Each carrier has demands from customers that constitute a commodity set, denoted by $\mathcal{K}_h$. For any commodity with a commodity-type serial number $k \in \{0, 1, \dots, |\mathcal{K}_h|\}$, it has the attribute: $\bar{d}_k^h, o_k^h, s_k^h, \sigma_k^h, \tau_k^h$, denoting nominal demand, the origin and destination, the departure time, and the deadline time, respectively, for the commodity k for carrier h . The above commodities can be distributed over a time-space extended network $\mathcal{G}(\mathcal{N}^t, \mathcal{A})$. $\mathcal{N}^t$ signifies a node set that can be serviced by the carriers h in time t and $o_k^h, s_k^h \in \mathcal{N}^t, \forall k, h$. For time t , when contemplating the commodity as a same-day or next-day delivery, it can be discretized into several equal time periods utilizing *time discretization* techniques (see, [Lium et al. \(2009\)](#), [Bai et al. \(2014\)](#), [Wang and Qi \(2020\)](#)). Set that the discretized time t belongs to the set $\mathcal{T} = \{0, 1, 2, \dots, |\mathcal{T}| - 1\}$ and $\sigma_k^h, \tau_k^h \in \mathcal{T}, \forall k, h$. Then the departure period for the freight truck arriving at the subsequent node at period t can be denoted as $t^- = t - 1$, if $t \geq 1$; $t^- = |\mathcal{T}| - 1$, otherwise. $\mathcal{A}$ includes the arc from i node to j node, $\forall i, j \in \mathcal{N}^t$. Carriers must make decisions $\mathcal{G}(\mathcal{N}^t, \mathcal{A})$ regarding the carrier that provides the service, the frequency of the service, and the distribution of the commodity flow in order to deliver the commodity in the most economically efficient transportation manner. These services are repeatedly executed during each *scheduling length* (a week) within the *planning horizon* (half a year or a season). Given that orders are shared among multiple parties, only one commodity set $\mathcal{K} = \bigcup_h \mathcal{K}_h$ needs to be completed and serial number $k \in \{0, 1, \dots, |\bigcup_h \mathcal{K}_h|\}$. Hence, the following is a simplified overview of the commodity properties of coalition transportation: $\bar{d}_k, o_k, s_k, \sigma_k, \tau_k$. It is important to note that an arbitrary carrier does not possess the authority to make decisions on behalf of all collaborative members. It is assumed that such decisions are rendered by the Intelligent Decision Support Platform or the Fourth-Party Logistics Platform (see [Taherkhani et al. \(2022\)](#) and [Giusti et al. \(2019\)](#)).

[Fig. 2](#) gives an instance of CSNDP in the order sharing model. The utilization of different lines and colors is applied to distinguish between freight routes and commodities, respectively. In the absence of cooperation, carrier 1 (solid line) would be required to transport commodity A independently, while carrier 2 would be responsible for transporting commodity B alone (refer to [Fig. 2\(a\)](#)). A total of three trucks are used. However, by sharing orders between carriers, they collaborate to fulfill commodity B, and the transportation task can be completed with only two trucks (as shown in [Fig. 2 \(b\)](#)). It is evident that the latter solution is more economical, given that vehicle movement between nodes incurs a fixed cost (e.g., employment of drivers).

To differentiate between the freight transportation capabilities of each carrier, the following parameters are defined: the fixed cost incurred by carrier h to provide service from node i to node j is denoted by f_{ijh} ; the maximum number of trucks that carrier h can accommodate is denoted by V_h ; and each truck has a capacity represented by u_h . Without loss of generality, outsourcing is allowed when the capacity of collaborating carriers cannot handle certain commodities. The unit outsourcing cost is denoted by λ .

Meanwhile, fluctuating customer demand is a common scenario in real logistics operations. The convenience of e-commerce platforms allows customers to change their orders or even return them for various reasons. Without the ability to handle fluctuating demand, logistics companies may face declining profit margins and customer satisfaction. It is therefore essential to incorporate demand uncertainty into CSNDP planning. In order to control the conservatism degree, we use a budget uncertainty set ([Bertsimas and Sim, 2004](#)) to characterize demand uncertainty. Assume that the nominal demand for the commodity k of carrier h , $\bar{d}_k^h$, and the potential maximum deviation, $\hat{d}_k^h$, are available. In order to hedge the risk of demand uncertainty, reliance is placed on consideration

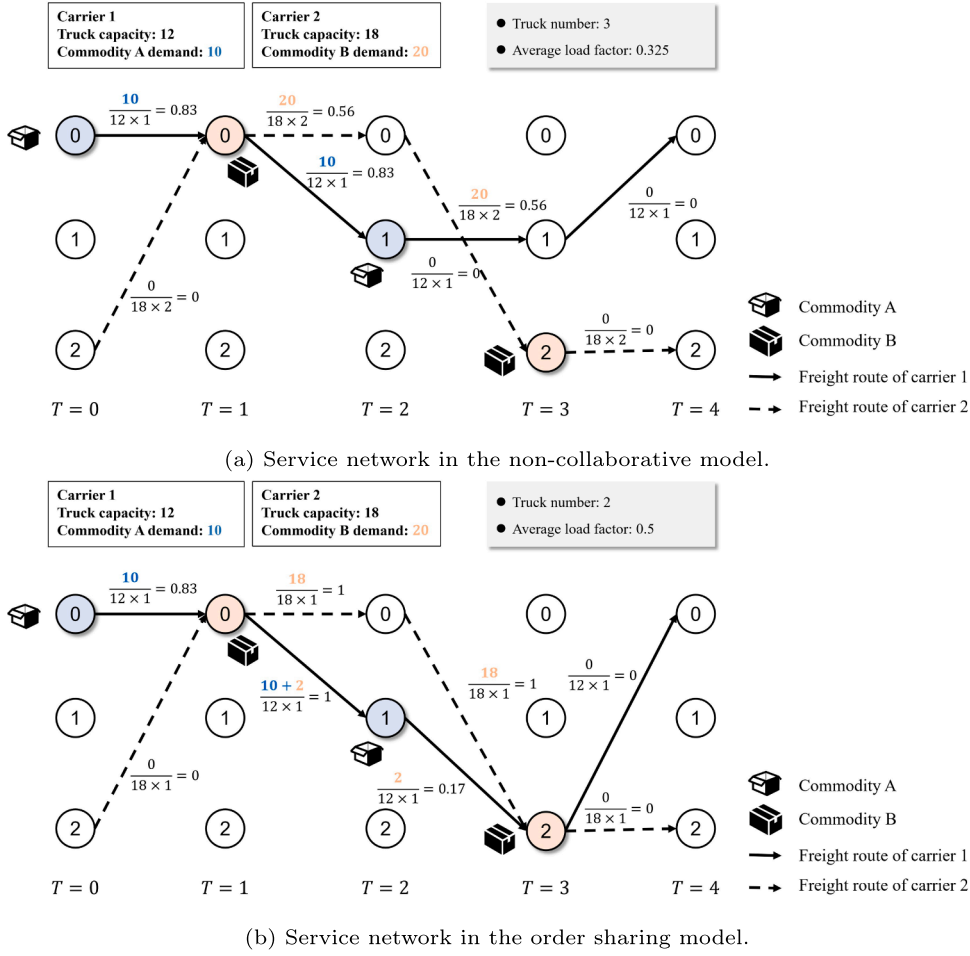


Fig. 2. The difference in the structure of the service network under non-collaboration and order sharing.

$d_k^h \in [\bar{d}_k^h, \bar{d}_k^h + \hat{d}_k^h]$. The uncertainty set is constructed as

$$\mathcal{U} = \left\{ d | d_k^h = \bar{d}_k^h + p_k^h \cdot \hat{d}_k^h, \sum_{k \in \mathcal{K}_h} p_k^h \leq \Gamma^h, 0 \leq p_k^h \leq 1, \forall k \in \mathcal{K}_h, \forall h \in \mathcal{H} \right\}, \quad (1)$$

where $\Gamma^h \in [0, |\mathcal{K}_h|]$ is the budget of uncertainty, which adjusts the robustness level for the carrier h . In instances where multiple carriers establish the coalition, it is necessary to evaluate the worst-case demand scenario at the collective coalition level, rather than concentrating solely on the individual carriers (see Zhang et al. (2024, 2025), Yin et al. (2026)). Hence, we assume that carriers collaborate in this process to undertake the risk caused by uncertain customer demands. Let the budget of the collaborative carriers be Γ . Then, the uncertainty set Eq. (1) can be expressed in a more concise form as follows:

$$\mathcal{U} = \left\{ d | d_k^h = \bar{d}_k^h + p_k^h \cdot \hat{d}_k^h, \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} p_k^h \leq \Gamma, 0 \leq p_k^h \leq 1, \forall k \in \mathcal{K}_h, \forall h \in \mathcal{H} \right\}$$

Based on the setup of the order sharing framework, we set the serial number $k \in \{0, 1, \dots, |\bigcup_h \mathcal{K}_h|\}$. Hence, there is $d_k = \bar{d}_k + p_k \cdot \hat{d}_k$, $\sum_{k \in \mathcal{K}} p_k \leq \Gamma, 0 \leq p_k \leq 1, \forall k \in \mathcal{K}$, where Γ indicates the risk budget.

The decision variables are denoted as follows.

- x_{ijh}^t : the service frequency provided by the carrier h between nodes i and j during the period t . $x_{ijh}^t \in \{0, 1, 2, \dots\}$.
- y_{ijh}^{kt} : the transportation volume of the commodity k transported by the carrier h between the nodes i and j during the period t . $y_{ijh}^{kt} \geq 0$.
- Z_k : the amount of outsourcing required for a commodity k in the optimal commodity flow. $Z_k \geq 0$.

Based on the uncertainty set Eq. (1), we formulate the CSNDP under the order sharing model into a two-stage robust optimization model, as follows:

$$\text{Stage 1} \quad \min_x \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} f_{ijh} \cdot x_{ijh}^t + \max_p \Omega(x, p) \quad (2a)$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{N}^t} x_{jih}^t = \sum_{j \in \mathcal{N}^t} x_{ijh}^t, \quad \forall i \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad (2b)$$

$$\sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t} x_{ijh}^t \leq V_h, \quad \forall t \in \mathcal{T}, \forall h \in \mathcal{H}, \quad (2c)$$

$$x_{ijh}^t \in \{0, 1, 2, \dots\}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \quad (2d)$$

$$\text{Stage 2} \quad \Omega(x, p) := \min_{y, Z} \sum_{k \in \mathcal{K}} \lambda \cdot Z_k \quad (2e)$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} y_{ijh}^{kt} \leq u_h \cdot x_{ijh}^t, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, i \neq j, \forall h \in \mathcal{H}, \quad (2f)$$

$$\begin{aligned} & \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} y_{ijh}^{kt} - \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} y_{jih}^{kt} \\ &= \begin{cases} \bar{d}_k + p_k \cdot \hat{d}_k - Z_k, & \text{if } i = o_k, t = \sigma_k \\ -\bar{d}_k - p_k \cdot \hat{d}_k + Z_k, & \text{if } i = s_k, t = \tau_k \\ 0, & \text{otherwise} \end{cases}, \forall i \in \mathcal{N}^t, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \end{aligned} \quad (2g)$$

$$\sum_{k \in \mathcal{K}} p_k \leq \Gamma, \quad (2h)$$

$$0 \leq p_k \leq 1, \quad \forall k \in \mathcal{K}, \quad (2i)$$

$$y_{ijh}^{k\tau_k} = 0, \quad \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}, \quad (2j)$$

$$y_{ijh}^{kt} \in \mathbb{R}^+, Z_k \in \mathbb{R}^+, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}. \quad (2k)$$

In the **Stage 1**, the objective function (2a) is designed to minimize the worst-case cost, which comprises the fixed cost and the outsourcing cost; Constraint (2b) is the asset-balancing equation. It ensures that the number of trucks entering and leaving each node for each carrier within the given period is balanced; Constraint (2c) restricts the maximum number of trucks each carrier invests in; In **Stage 2**, Constraint (2f) serves as a capacity constraint, ensuring that the total flow does not exceed the network capacity; Constraint (2g) ensures the flow conservation equations for each commodity and every node at each period. It should be noted that the aforementioned summation over h guarantees the order's splitting among carriers; Constraint (2h) and Constraint (2i) portrays uncertainty set; Constraint (2j) ensures that each commodity can reach its destination before the deadline; Constraint (2d) and Constraint (2k) define the domain of the decision variables. Call the model **OS-TSRO**.

3.2. Vehicle sharing

Vehicle sharing is another collaborative framework. Within this framework, there are basic assumptions (see Sahand et al. (2024)): (1) Multiple collaborative carriers; (2) Each carrier has customer orders to serve and its own vehicle fleets; (3) Collaborative carriers cannot directly accomplish each other's orders; (4) Carriers can use each other's vehicles; and (5) All nodes are accessible to every carrier h . The main difference in comparison to order sharing is that commodities cannot be fulfilled by multiple carriers. That is, each carrier independently uses its own or leased vehicles to satisfy its own customer demands $\mathcal{K}_h$. An additional decision variable $V_{hh'}$ needs to be introduced, denoting the number of vehicles leased by carrier h to h' . Meanwhile, the outsourcing volume of the optimal commodity flow for each carrier is denoted as Z_k^h . Then, the collaborative carriers address a CSNDP on the graph $\mathcal{G}(\mathcal{N}^t, \mathcal{A})$ in the framework of vehicle sharing. Refer to Section 3.1 for other settings.

Fig. 3 gives an instance of CSNDP in the vehicle sharing model. As shown in Fig. 3(a), in the case of non-collaboration, there is an order that cannot be fully fulfilled due to capacity constraints of carrier 1 (solid red line). In the event that Carrier 1 is to fulfill this order, it would be necessary for them to engage in expensive outsourcing. In contrast, under the vehicle sharing model, carrier 2 lends carrier 1 an unused vehicle to fulfill the order, which means gaining higher profits and avoiding outsourcing. In this framework, the order is not exchanged. Additionally, prior to the presentation of the CSNDP formula for vehicle sharing, a necessary issue is to determine whether leasing a vehicle involves additional vehicle scheduling costs. For instance, the driver of carrier 2 arrives at node 2 ahead of schedule. In order to circumvent the possibility of decision-making processes being driven by individual self-interest, resulting in the excessive borrowing of vehicles from other carriers, we set the dispatching cost of borrowing a vehicle to μ .

In the context of customer trust, it is imperative that leased vehicles be pre-treated, such as through placing labels bearing the lessee's identification on the leased vehicle. We assume that the vehicles borrowed by the carrier h from h' ($h' \neq h$) have the same capacity u_h . In accordance with the parameters and decision variables delineated in section 3.1, the CSNDP formula for vehicle sharing is hereby developed as follows:

$$\text{Stage 1} \quad \min_x \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} f_{ijh} \cdot x_{ijh}^t + \sum_{h \in \mathcal{H}} \sum_{h' \in \mathcal{H} \& h' \neq h} \mu \cdot V_{hh'} + \max_p \Omega(x, p) \quad (3a)$$

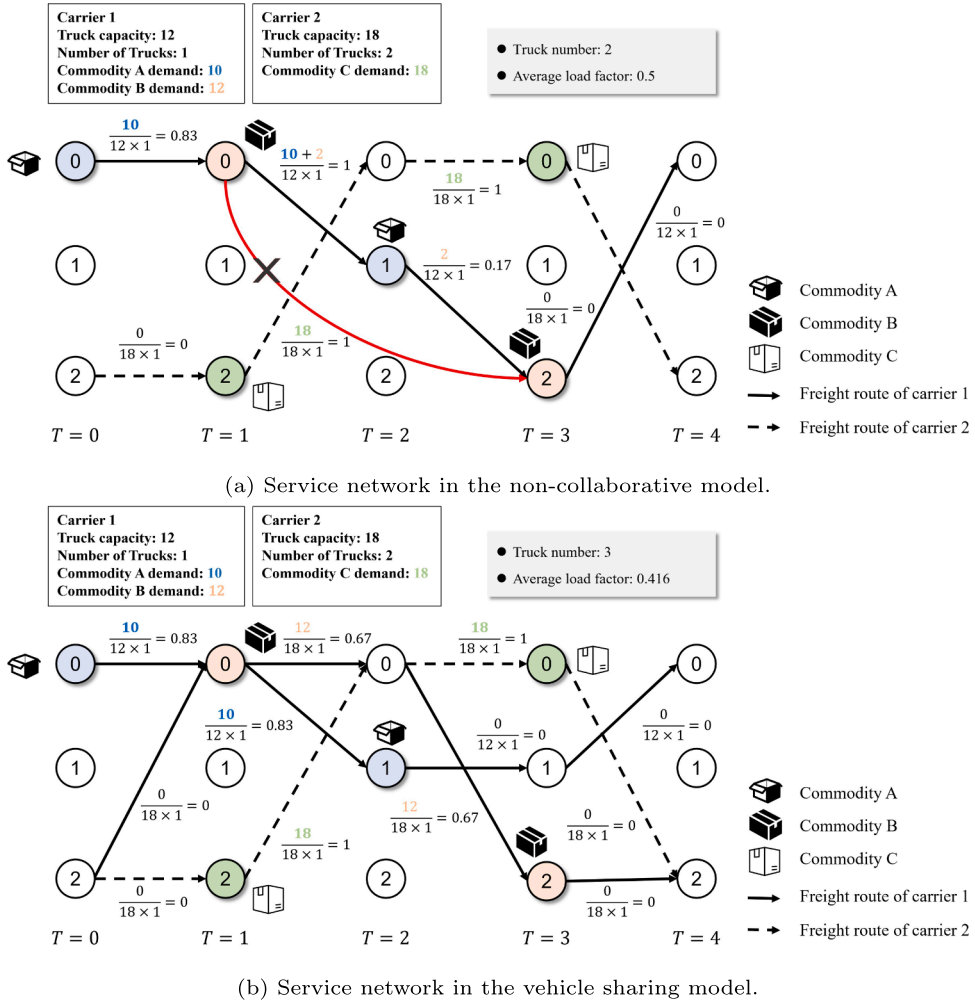


Fig. 3. The difference in the structure of the service network under non-collaboration and vehicle sharing.

$$\text{s.t. } \sum_{j \in \mathcal{N}^t} x_{jih}^{t-} = \sum_{j \in \mathcal{N}^t} x_{ijh}^t, \quad \forall i \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad (3b)$$

$$\sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t} x_{ijh}^t \leq V_h + \sum_{h' \in \mathcal{H} \& h' \neq h} V_{h'h} - \sum_{h' \in \mathcal{H} \& h' \neq h} V_{hh'}, \quad \forall t \in \mathcal{T}, \forall h \in \mathcal{H}, \quad (3c)$$

$$x_{ijh}^t \in \{0, 1, 2, \dots\}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \quad (3d)$$

$$V_{hh'} \in \{0, 1, 2, \dots\}, \quad \forall h, h' \in \mathcal{H} \quad h' \neq h, \quad (3e)$$

$$\text{Stage 2 } \Omega(x, p) := \min_{y, Z} \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} \lambda \cdot Z_k^h \quad (3f)$$

$$\text{s.t. } \sum_{k \in \mathcal{K}_h} y_{ijh}^{kt} \leq u_h \cdot x_{ijh}^t, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, i \neq j, \forall h \in \mathcal{H}, \quad (3g)$$

$$\sum_{j \in \mathcal{N}^t} y_{ijh}^{kt} - \sum_{j \in \mathcal{N}^t} y_{jih}^{kt-} = \begin{cases} \bar{d}_k^h + p_k^h \cdot \hat{d}_k^h - Z_k^h, & \text{if } i = o_k^h, t = \sigma_k^h \\ -\bar{d}_k^h - p_k^h \cdot \hat{d}_k^h + Z_k^h, & \text{if } i = s_k^h, t = \tau_k^h \\ 0, & \text{otherwise} \end{cases}, \quad \forall i \in \mathcal{N}^t, \forall k \in \mathcal{K}_h, \forall t \in \mathcal{T}, \forall h \in \mathcal{H}, \quad (3h)$$

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} p_k^h \leq \Gamma, \quad \forall h \in \mathcal{H}, \quad (3i)$$

$$0 \leq p_k^h \leq 1, \quad \forall k \in \mathcal{K}_h, \forall h \in \mathcal{H}, \quad (3j)$$

$$y_{ijh}^{k\tau h} = 0, \quad \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (3k)$$

$$y_{ijh}^{kt} \in \mathbb{R}^+, Z_k \in \mathbb{R}^+, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h. \quad (3l)$$

Constraint (3c) restricts the maximum number of vehicles each carrier can commit and allows carriers to lease vehicles to each other; Constraint (3g) indicates that after a leased vehicle operation has been executed, the carrier commodity flow must not exceed the network capacity; Constraint (3h) ensures that commodities can flow between nodes. However, it is important to note that orders are not permitted to be fulfilled in a collaborative manner. Constraint (3i) indicates that the budget is set centrally by the co-carrier. For further information regarding other constraint roles, please refer to model (2a). Call the model **VS-TSRO**.

3.3. Summary of models

In summary, we propose two two-stage robust optimization models (OS-TSRO and VS-TSRO) for CSNDP. The mechanism underpinning these two collaborative models is distinct. **OS-TSRO** facilitates the collaborative transportation of orders and the optimization of global costs by sharing orders; **VS-TSRO** ensures the confidentiality of customer information and an increase in profits by empowering carriers to share idle vehicles.

There are also some connections between the two models. Equivalent solutions may emerge under the two frameworks when the homogenization of the carriers' assets in the coalition or the splitting of orders does not enhance economic efficiency. In Section 5, we undertake a comparative analysis of the two models' solutions in various practical scenarios and offer some insights into the process of selection.

4. SDS-CCG method

Both **OS-TSRO** and **VS-TSRO** are known as TSRO models, which are recognized as NP-hard (refer to Ben-Tal and Goryashko (2004)). In response to this complexity, researchers have developed various solution algorithms. Among these, the column-and-constraint generation algorithm has emerged as one of the state-of-the-art approaches. Research conducted by Wang and Qi (2019, 2020), Zhang et al. (2024) has demonstrated that the algorithm significantly improves computational efficiency, outperforming the classical Benders decomposition algorithm. Nonetheless, it is important to note that the proposed model is more difficult to solve due to its more complex logic and larger scale. To address this challenge, this paper proposes a novel SDS-CCG algorithm. The SDS-CCG algorithm identifies certain critical scenarios in advance by embedding a pre-trained classifier, thereby facilitating accelerated convergence. The subsequent sections will provide a detailed exposition of the proposed solution methodology.

4.1. Train the classifier

Before outlining the methodology for training a classifier, it is essential to discuss the reason behind utilizing the classifier. The model engages in CSNDP planning based on a budget uncertainty set, which facilitates the development of robust solutions. As illustrated in Eq. (1), for a budget uncertainty set containing $|\mathcal{H}|$ carriers with $|\mathcal{K}_h|$ commodities and $\Gamma = a$, the number of extreme points is the combination number $C_{\sum_{h \in |\mathcal{H}|} |\mathcal{K}_h|}^a$. An increase in the number of commodities or carriers will result in a combinatorial explosion in the number of possible combinations. It is imperative to recall the operational process of the column-and-constraint generation algorithm, wherein the subproblem identifies the worst-case scenario based on the current design decisions $\mathbf{x}$ made in the master problem during each iteration and adds relevant resource variables and constraints about the worst-case scenario to the master problem until convergence is achieved (Zeng and Zhao, 2013). The term "current" means a short-sighted approach; after selecting a "worst-case" scenario from a multitude of extreme combinations within the budget uncertainty set, the algorithm lacks the capacity to foresee the implications of this generation on the efficiency of subsequent solutions. That is, some added scenarios may have a negligible impact on the convergence of the overall algorithm. However, this results in a continuous scaling up of the master problem, with solution times growing exponentially. Therefore, our first step is to train a classifier to help the algorithm identify some potential worst-case scenarios that are conducive to convergence in advance. In Section 4.1.1, we detail the construction methods for the dataset; in Section 4.1.2, the specific form of classifier is introduced.

4.1.1. Construction of the dataset

Before training the classifier, the dataset must be constructed. The dataset should contain demand data that has been determined to represent a worst-case scenario or not. Typically, when the overall freight demand surpasses the service network capacity, carriers may resort to temporary scheduling measures to fulfill delivery requirements, a practice commonly referred to as outsourcing. Consequently, the carrier could assess whether a given demand scenario constitutes a potential worst-case scenario by evaluating the associated outsourcing costs. When demand data is sufficient, the carrier could utilize this historical information to develop and train a classification model. To overcome the issue of limited data availability, this section introduces a methodology for training classifiers utilizing synthetic datasets.

For the budget uncertainty set, each uncertain parameter combination indicates a demand scenario faced by the carriers. For example, $\{\hat{d}_0^0 + p_0^0 \cdot \hat{d}_0^0, \dots, \hat{d}_{|\mathcal{K}_0|-1}^0 + p_{|\mathcal{K}_0|-1}^0 \cdot \hat{d}_{|\mathcal{K}_0|-1}^0, \dots, \hat{d}_{|\mathcal{K}_{|\mathcal{H}|-1}|-1}^{|\mathcal{H}|-1} + p_{|\mathcal{K}_{|\mathcal{H}|-1}|-1}^{|\mathcal{H}|-1} \cdot \hat{d}_{|\mathcal{K}_{|\mathcal{H}|-1}|-1}^{|\mathcal{H}|-1}\}$. However, it is impractical to enumerate all scenarios exhaustively within a budget uncertainty set. For the sake of convenience, 1,000 pieces of demand data d_s have been randomly generated within the budget set, which is the input to our classifier. Meanwhile, for every demand scenario d_s , it is

imperative to assign a label. We use label 1 to signify that the scenario s may be a potential worst-case scenario and label -1 to indicate that it may not fall into this category. The label-setting process is conducted in accordance with the following algorithm.

Algorithm 1: The label-setting algorithm.

Input: The uncertain scenario $d_s \in S$, S denotes the generated demand scenario set.

- 1 Define the threshold θ to denote the maximum outsourcing cost that carriers can tolerate.
- 2 The collaborative carriers solve the CSNDP by utilizing the expected values of all commodities $\bar{d}_k^h$, thereby deriving the decision $\bar{x}$ in a deterministic environment.
- 3 **while** $d_s \in S$ **do**
- 4 Fix the decision $\bar{x}$, re-evaluate the performance of $\bar{x}$ under scenario d_s , and observe outsourcing cost $Cost_{out}$.
- 5 If $Cost_{out} \geq \theta$, setting label to 1; -1, otherwise.
- 6 **end**

Output: The label 1/-1 of the demand scenario $d_s \in S$

According to Algorithm 1, we obtain the dataset $\{d_s, 1/-1\}$ for training. Note that the value of the parameter θ has a direct impact on the distribution of positive and negative samples within the dataset. Specifically, a big threshold θ will result in a greater proportion of negative samples, leading to disregarding scenarios that may be beneficial to the convergence. Conversely, a small θ will result in a classifier with limited capacity to reduce the scenario space and an inability to accelerate algorithm convergence. This suggests that SDS-CCG performs optimization based on a subset of the uncertainty set Eq. (1), which may lead to failure to achieve an exact optimal solution. Nonetheless, the outsourcing strategy makes the second-stage problem satisfy the complete recourse nature, which ensures the solutions obtained by the algorithm are always feasible for the two-stage robust optimization model.

4.1.2. Definition of a classifier

Common classifiers include linear classification, support vector machines, and decision trees. In this paper, three classifiers are trained and tested using the dataset constructed in the manner of the previous section. The accuracy and positive class recall are used as evaluation metrics, and the results are shown in the following table.

As demonstrated in Table 2, there is not an obvious discrepancy in classification accuracy among the three classifiers. The linear classifier's simple structure facilitates its integration within the optimization model while exhibiting a positive class recall that exceeds 90%. In this paper, the linear classifier is employed. The form of the classifier is as follows:

$$\kappa(p_k^h) = \begin{cases} 1, & \text{if } \sum_h \sum_k (\bar{d}_k^h + p_k^h \hat{d}_k^h) \cdot \rho_k^h + b \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

where ρ_k^h and b represent the weights and bias terms, respectively, of the trained linear classifier.

Linear classification identifies whether the scenario d_s defined by any combination p_k^h in the budget set is a potential worst-case scenario. In the event that this is not the worst case, the scenario is excluded from the optimization process. It should be noted that, due to the limitations of computer precision, it is frequently difficult to execute the aforementioned calculations such that the scenario labels correspond precisely to the value of 1/-1. The following representation is employed in the model:

$$\sum_h \sum_k (\bar{d}_k^h + p_k^h \hat{d}_k^h) \cdot \rho_k^h + b \geq 0$$

For the order sharing framework, the above equation can be simplified as

$$\sum_k (\bar{d}_k + p_k \hat{d}_k) \cdot \rho_k + b \geq 0$$

4.2. Master problem and sub-problem for CSNDP

According to the process of column-and-constraint generation algorithms (Zeng and Zhao, 2013), the TSRO model is decomposed into two parts, where a master problem (MP) goes to decision-making when uncertain information is not revealed and a subproblem (SP) is developed to evaluate the worst-case operational cost under a given decision. In this paper, we define CSNDP as the MP to determine the network structure x and reformulate the SP to evaluate the worst-case outsourcing cost under decision x . Simultaneously, the SP embeds a linear classifier defined in Section 4.1.2 for downscaling the number of scenarios. The reformulation of OS-TSRO is provided in the following section, with VS-TSRO detailed in Appendix A.

Table 2
Performance of different classifiers.

Classifier	Accuracy rate	Recall rate
Linear classification	95.1%	91.2%
Support vector machine	96.3%	89.5%
Decision tree	85.4%	90.6%

4.2.1. OS-SP

First, we employ the primal-dual method to derive the solvable dual reformulation of the second-stage inner OS-TSRO model. Given any fixed $(\bar{x}, \bar{p})$, the constraints (2f), (2g) and (2j) are handled through the utilization of dual variables, α , β , and γ , respectively. Then the dual problem of the inner problem OS-DRP($\bar{x}, \bar{p}$) is given as

$$(\text{OS-DRP}(\bar{x}, \bar{p})) \quad \max_{\alpha, \beta, \gamma} - \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t \setminus \{i\}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} u_h \bar{x}_{ijh}^t \alpha_{ijh}^t + \sum_k \left(\beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k} \right) \left(\bar{d}_k + \bar{p}_k \hat{d}_k \right) \quad (4a)$$

$$\text{s.t.} \quad \beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k} \leq \lambda, \quad \forall k \in \mathcal{K}, \quad (4b)$$

$$\beta_i^{kt^-} - \beta_j^{kt} \leq \begin{cases} \alpha_{ijh}^{t^-}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}, \quad \forall t \in \mathcal{T}, \forall t^- \in \mathcal{T} \setminus \{\tau_k\}, \forall i, j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}, \quad (4c)$$

$$\beta_i^{k\tau_k} - \beta_j^{kt} + \gamma_{ijh}^k \leq \begin{cases} \alpha_{ijh}^{t\tau_k}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}, \quad \forall t \in \mathcal{T}, \forall i, j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}, \quad (4d)$$

$$\alpha_{ijh}^t \geq 0, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \quad (4e)$$

$$\sum_{k \in \mathcal{K}} \bar{p}_k \leq \Gamma, \quad (4f)$$

$$0 \leq \bar{p}_k \leq 1, \quad \forall k \in \mathcal{K}, \quad (4g)$$

$$\sum_k (\bar{d}_k + \bar{p}_k \hat{d}_k) \cdot \rho_k + b \geq 0, \quad (4h)$$

Subsequent to the aforementioned reformulation, the second stage can be converted into a single-layer maximization problem:

$$(\text{OS-DRP}(\bar{x})) \quad \max_{\alpha, \beta, \gamma, p} - \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t \setminus \{i\}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} u_h \bar{x}_{ijh}^t \alpha_{ijh}^t + \sum_k \left(\beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k} \right) \left(\bar{d}_k + p_k \hat{d}_k \right) \quad (5a)$$

$$\text{s.t.} \quad \text{Constraints } 4b - 4e$$

$$\sum_{k \in \mathcal{K}} p_k \leq \Gamma, \quad (5b)$$

$$0 \leq p_k \leq 1, \quad \forall k \in \mathcal{K}, \quad (5c)$$

$$\sum_k (\bar{d}_k + p_k \hat{d}_k) \cdot \rho_k + b \geq 0, \quad (5d)$$

Note that the second term $\sum_k \left(\beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k} \right) p_k$ in OS-DRP($\bar{x}$) has nonlinearity. According to Wang and Qi (2020), there exists an optimal solution for OS-DRP($\bar{x}$) and $p_k \in \{0, 1\}$. Thus, we provide a linear reformulation of the model by introducing an auxiliary decision variable w_k . Define

$$w_k = \begin{cases} \beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k}, & \text{if } p_k = 1 \\ 0, & \text{if } p_k = 0 \end{cases}$$

Subsequently, the OS-SP is introduced in the following formulation:

$$(\text{OS-SP}) \quad \max_{\alpha, \beta, \gamma, p} - \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t \setminus \{i\}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} u_h \bar{x}_{ijh}^t \alpha_{ijh}^t + \sum_k \left(\beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k} \right) \cdot \bar{d}_k + \sum_k \hat{d}_k w_k \quad (6a)$$

$$\text{s.t.} \quad \text{Constraints } 4b - 4e$$

$$w_k \leq \beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k}, \quad \forall k \in \mathcal{K}, \quad (6b)$$

$$w_k + (1 - p_k) \cdot M \geq \beta_{o_k}^{k\sigma_k} - \beta_{s_k}^{k\tau_k}, \quad \forall k \in \mathcal{K}, \quad (6c)$$

$$w_k \leq M \cdot p_k, \quad \forall k \in \mathcal{K}, \quad (6d)$$

$$w_k \geq 0, \quad \forall k \in \mathcal{K}, \quad (6e)$$

$$\sum_{k \in \mathcal{K}} p_k \leq \Gamma, \quad (6f)$$

$$p_k \in \{0, 1\}, \quad \forall k \in \mathcal{K}, \quad (6g)$$

$$\sum_k (\bar{d}_k + p_k \hat{d}_k) \cdot \rho_k + b \geq 0, \quad (6h)$$

where M is a large constant. 6b - 6h ensures that the linearized model is equivalent to the original model. OS-SP defines the subproblem of identifying the worst-case scenario under given x .

4.2.2. OS-MP

OS-MP is not initially in possession of any information regarding the uncertainty. Given x , solve the OS-SP defined in Section 4.2.1 to obtain the worst-case scenario under the current decision and add the scenario to the OS-MP. Subsequently, the OS-MP makes

decisions in response to the uncertainty scenarios that have been identified. This process is to be repeated until the OS-SP is no longer able to identify new scenarios that can be added to the OS-MP. The decision at this juncture is tantamount to the OS-TSRO near-optimal solution. Let η to represent an evaluation of the worst-case outsourcing cost, the OS-MP is given as

$$\text{(OS-MP)} \quad \min_x \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} f_{ijh} \cdot x_{ijh}^t + \eta \quad (7a)$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{N}^t} x_{jih}^t = \sum_{j \in \mathcal{N}^t} x_{ijh}^t, \quad \forall i \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad (7b)$$

$$\sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t} x_{ijh}^t \leq V_h, \quad \forall t \in \mathcal{T}, \forall h \in \mathcal{H}, \quad (7c)$$

$$x_{ijh}^t \in \{0, 1, 2, \dots\}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \quad (7d)$$

$$\eta \geq \sum_k (Z_k)^l, \quad \forall l = 1, \dots, L, \quad (7e)$$

$$\sum_{k \in \mathcal{K}} (y_{ijh}^{kt})^l \leq u_h \cdot x_{ijh}^t, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, i \neq j, \forall h \in \mathcal{H}, \forall l = 1, \dots, L, \quad (7f)$$

$$\begin{aligned} & \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} (y_{ijh}^{kt})^l - \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} (y_{jih}^{kt})^l \\ &= \begin{cases} \bar{d}_k + (p_k^*)^l \cdot \hat{d}_k - (Z_k)^l, & \text{if } i = o_k, t = \sigma_k \\ -\bar{d}_k - (p_k^*)^l \cdot \hat{d}_k + (Z_k)^l, & \text{if } i = s_k, t = \tau_k \\ 0, & \text{otherwise} \end{cases}, \quad \forall i \in \mathcal{N}^t, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, l = 1, \dots, L, \end{aligned} \quad (7g)$$

$$\sum_{k \in \mathcal{K}} (p_k^*)^l \leq \Gamma, l = 1, \dots, L, \quad (7h)$$

$$(p_k^*)^l \in \{0, 1\}, \quad \forall k \in \mathcal{K}, l = 1, \dots, L, \quad (7i)$$

$$\sum_k (\bar{d}_k + (p_k^*)^l \hat{d}_k) \cdot \rho_k + b \geq 0, \quad l = 1, \dots, L, \quad (7j)$$

$$(y_{ijh}^{kt_k})^l = 0, \quad \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}, l = 1, \dots, L, \quad (7k)$$

$$(y_{ijh}^{kt})^l \in \mathbb{R}^+, (Z_k)^l \in \mathbb{R}^+, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}, l = 1, \dots, L, \quad (7l)$$

where $(p_k^*)^l$ is the worst-case scenario identified by **OS-SP** for the l th iteration. 7e–7l are resource constraints.

4.3. The SDS-CCG algorithm

Within the SDS-CCG framework, initially, the linear classifier is trained for the purpose of identifying critical scenarios. Then, we would achieve the solution by iteratively solving OS-SP and OS-MP. We provide the pseudocode of the SDS-CCG algorithm as follows (Algorithm 2).

Algorithm 2: The SDS-CCG algorithm.

Input: All parameters and sets in CSNDP problem; termination condition ϵ

- 1 Initialize iteration $l = 1$, upper bound $UB = \infty$, lower bound $LB = 0$.
- 2 Training a linear classifier $\kappa(\cdot)$.
- 3 Randomly initialize $\tilde{x}$, solve **OS-SP/VS-SP** to obtain the worst-case $(p_k^*)^l$ and optimal objective function value sp^l .
- 4 **while** $(UB - LB)/UB > \epsilon$ **do**
- 5 Recurse variables and constraints are added to **OS-MP/VS-MP**. Then solve **OS-MP/VS-MP** to obtain x^l , optimal objective function value mp^l and update low bound $LB = \max\{LB, mp^l\}$.
- 6 Based on x^l to solve **OS-SP/VS-SP**, and obtain the worst-case $(p_k^*)^l$ and optimal objective function value sp^l . Update upper bound $UB = \min\{UB, f \cdot x^l + sp^l\}$.
- 7 $l = l + 1$.
- 8 **end**

Output: An CSNDP scheme x

5. Numerical study

The numerical experiments are implemented in Python based on the PyCharm compiler, using Gurobi 11.0.3 as the mixed integer programming solver. All experiments were conducted on an Intel(R) Core(TM) i5-14400F processor, operating at 2.50 GHz and supported by 16 GB of RAM and an NVIDIA GeForce RTX 4060 Ti.

Table 3
Parameter settings.

Parameter	value	Parameter	value
$ \mathcal{H} $	[2, 3]	f_{ijh}	$U \sim (100, 200)$
$ \mathcal{N}^i $	[6, 12]	u_h	$U \sim (15, 25)$
$ \mathcal{K}_h $	[4, 8]	λ	150
$ \mathcal{T} $	[5, 10]	$\hat{d}_k^h$	10

Table 4
Performance of SDS-CCG and column-and-constraint generation algorithm.

Instance	OS-TSRO							VS-TSRO						
	SDS-CCG			CCG				SDS-CCG			CCG			
	CPU	UB	GAP	CPU	UB	GAP	GAP^*	CPU	UB	GAP	CPU	UB	GAP	GAP^*
H2K4N6T5	1	2337	0	1	2337	0	0	1	2847	0	1	2847	0	0
H2K4N6T10	1	2412	0	3	2412	0	0	1	2627	0	1	2627	0	0
H2K4N12T5	12	1969	0	21	1969	0	0	2	2475	0	3	2475	0	0
H2K4N12T10	726	2505	0	1097	2487	0	0.7%	241	3504	0	316	3504	0	0
H2K8N6T5	20	4469	0	96	4469	0	0	1	5761	0	1	5761	0	0
H2K8N6T10	986	3895	0	1354	3895	0	0	2	5982	0	4	5982	0	0
H2K8N12T5	466	3727	0	652	3727	0	0	1	4060	0	1	3944	0	2.8%
H2K8N12T10	3877	3844	0	4959	3844	0	0	806	5001	0	1157	5001	0	0
H3K4N6T5	1	3162	0	1	3162	0	0	1	4707	0	1	4707	0	0
H3K4N6T10	639	2776	0	858	2776	0	0	2	4959	0	3	4959	0	0
H3K4N12T5	329	3594	0	557	3512	0	2.2%	1	4181	0	1	4181	0	0
H3K4N12T10	3177	3638	0	4379	3525	0	3.1%	296	5836	0	375	5836	0	0
H3K8N6T5	1124	4494	0	1531	4494	0	0	2	7850	0	3	7850	0	0
H3K8N6T10	3159	5496	0	3953	5496	0	0	61	7554	0	134	7554	0	0
H3K8N12T5	2754	5957	0	3116	5957	0	0	13	7343	0	19	7343	0	0
H3K8N12T10	9165	7251	20.9%	12505	7564	24.1%	-	1144	7632	0	1578	7632	0	0
Average	1652	3845	1.3%	2192	3852	2%	0.3%	161	5145	0	225	5138	0	0.1%

5.1. Experimental setup

The purposes of the numerical experiments are threefold: (i) to conduct extensive numerical experiments to verify the effectiveness of the proposed algorithm. (ii) to analyze the robustness of robust model solutions. (iii) to investigate the performance and solution characteristics of the two collaborative frameworks. Furthermore, to provide valuable managerial insights for freight service planners, we also visualized the differences between solutions under various collaborative modes and conducted a qualitative analysis of the solution structure based on small-scale instances. In this section, based on (Lium et al., 2009; Wang and Qi, 2020), we generated 16 group instances. Table 3 offers a detailed description about the ranges of experimental parameters. Various combinations of parameters generate distinct test instances.

As illustrated in Table 3, the generation rules for instance sizes are located on the left-hand side of the table, encompassing a total of 16 groups of instances. All collaborative carriers hold an equal number of orders. Each instance is named using its structure parameters; for example, instance H2K8N6T5 indicates there are 2 carriers, 8 commodities, 6 transport nodes, and 5 time periods. Furthermore, 'high' and 'low' are utilized to express the risk of demand fluctuations. We set the customer's actual demand d_k^h to fall into $U \sim (10, 20)$ for low risk and $U \sim (20, 30)$ for high risk, respectively. Correspondingly, freight transportation planners planning CSNDP under high/low uncertain degrees will set demand deviation $\hat{d}_k^h \in U \sim (0, 10)$ and $\hat{d}_k^h \in U \sim (10, 20)$, respectively. Finally, to ensure fairness, the leasing vehicle dispatch cost μ is set as a small constant.

5.2. Algorithm performance analysis

In this section, an analysis is conducted on the performance of the proposed SDS-CCG algorithm. The benchmark algorithm that is used for comparison is the column-and-constraint generation (CCG) algorithm. We evaluate the performance of both algorithms using the following performance metrics: 1) CPU time: The CPU time spent by both algorithms to solve CSNDP is limited to a maximum of 7200 seconds. Note that if the time limit is exceeded, the algorithm will stop after this iteration concludes rather than immediately; 2) Best UB: The optimal upper bound that was attained at algorithm termination; 3) GAP: The disparity between the upper and lower bound of the output at algorithm termination, denoting $GAP = (UB - LB)/UB * 100\%$. Furthermore, the decision of SDS-CCG may not be equal to the optimal solution due to the fact that the linear classifier may ignore a part of the important scenarios. We denote $GAP^* = (UB_S - UB_C)/UB_S * 100\%$ to represent the loss of precision, where UB_S and UB_C are the upper bounds obtained by the SDS-CCG and column-and-constraint generation algorithm, respectively. The average performance of the two algorithms for solving 16 different group instances is reported in Table 4.

As demonstrated in Table 4, the proposed algorithm exhibits superior performance in terms of solution speed compared to the CCG algorithm, while demonstrating only a small loss in accuracy. Specifically, both the proposed algorithm and the CCG have the

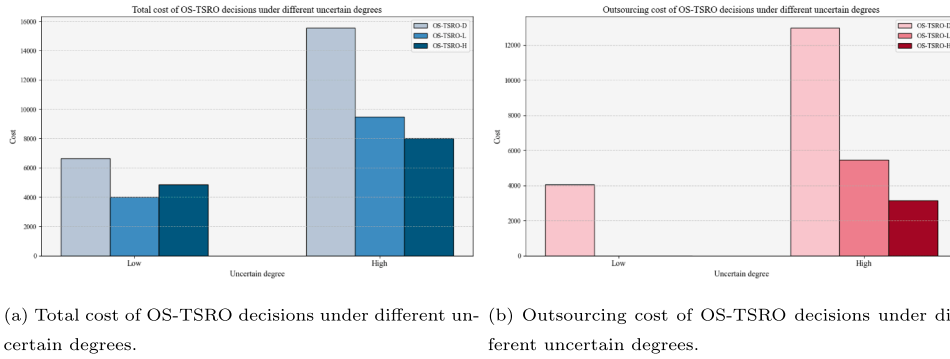


Fig. 4. Decision robustness analysis of OS-TSRO models under different uncertain degrees.

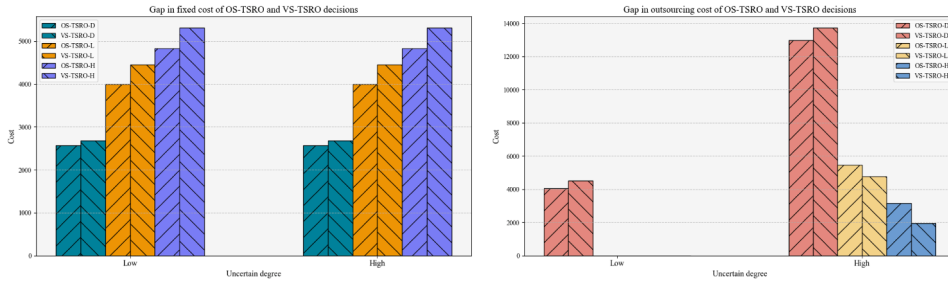
capacity to achieve an optimal solution for most instances within the set time limits. However, the SDS-CCG algorithm has been shown to be capable of solving instances of both collaborative frameworks in an average solving time of 1813 seconds. This is a 25% reduction in the average solving time of 2417 seconds for CCG. It is evident from the observation of the GAP^* column that the SDS-CCG algorithm exhibits only a marginal decline in accuracy. This is almost comparable to the original problem's optimal solution. The rationale behind this methodology is that the SDS-CCG algorithm can identify scenarios conducive to convergence in advance, thereby avoiding adding less significant scenarios in the MP and optimizing solution efficiency. This is particularly relevant in cases of large problem sizes, where solving the MP once can be challenging, and so it is necessary to ensure that the recourse constraints contribute positively to convergence. However, it is important to acknowledge that the SDS-CCG algorithm is not always effective, although we have not encountered this situation. Indeed, if the classifier overlooks a critical scenario, it may become necessary for the SDS-CCG to identify additional scenarios to mitigate this adverse impact, which can subsequently hinder the algorithm's convergence rate. Therefore, the performance of the classifier is crucial to the overall effectiveness of the algorithm.

Furthermore, it is evident that the solution time for OS-TSRO is occasionally approximately an order of magnitude greater than that of VS-TSRO. This difference stems from the fact that the OS-TSRO is responsible for centrally planning all orders owned by carriers and assigning these orders to the most appropriate carriers for delivery. In contrast, the VS-TSRO model, even in the case of instance H3K8N12T10, can be likened to solving three distinct H1K8N12T10 problems independently, albeit with the added complexity of vehicle leasing among the collaborating carriers. This comparison underscores the benefits of the vehicle sharing framework, particularly in its capacity to optimize computational resource utilization. Analogous conclusions are found in the study by Sahand et al. (2024). Finally, disregarding the variations in actual demand, the decision costs incurred by OS-TSRO are invariably lower. This analysis will be expanded upon in a subsequent section. In conclusion, the SDS-CCG algorithm has been shown to demonstrate superior performance regarding solution speed in the test instances.

5.3. Robustness analysis

In this section, to explore how both collaborative frameworks are flexible in dealing with uncertain demands, we analyze the solution robustness under two collaborative frameworks. All experiments were performed on instance H2K4N6T5. The models used as benchmarks for the comparison are deterministic models under two collaborative frameworks (i.e., $\hat{d}_k^h = 0, \Gamma = 0$). The OS-TSRO and VS-TSRO models are solved for using the maximum value of the demand deviation and $\Gamma = 5$ under different uncertain degrees. Specifically, the models are to be solved with different demand deviations under different collaborative frameworks to obtain the decision x , fix x and use the 500 test demand data that generate from the interval corresponding to the demand risk to assess the decision robustness. And we use the average total cost as the evaluation metric. In the interest of impartiality, all models are faced with the same commodity sets. The relevant results are shown in the following figure. Note that the decision-making performances of OS-TSRO under the deterministic settings, demand deviation $\hat{d}_k^h = 10$, and demand deviation $\hat{d}_k^h = 20$ are to be respectively named as OS-TSRO-D, OS-TSRO-L, and OS-TSRO-H. The “high” and “low” on the horizontal axis represent the actual demand risk levels. The same applies to VS-TSRO.

Fig. 4 illustrates the variations in the total freight transportation cost and outsourcing cost under different degrees of uncertainty within the OS-TSRO model. As shown in Fig. 4(a), an increase in total transportation cost correlates with a rise in practical demands. This is because the cost associated with transporting a specific freight volume is directly proportional to the overall quantity transported. However, it is important to note that this enhancement in costs is not solely attributable to the asset inputs from companies. Fig. 4(b) shows that when faced with high risk in demand, decisions made under demand deviation $\hat{d}_k^h \in U \sim (0, 10)$ can lead to considerable outsourcing costs, particularly in deterministic solutions. This situation arises from an inadequate consideration to account for demand uncertainty during the planning phase. As a result, inadequate investment in assets hampers the ability to mitigate the risks associated with demand variability during the operational phase. It is evident that the negative repercussions on logistics companies are substantial and emphasize the necessity for robustness. In Section 5.4, we visualize the deterministic and robust solution structure based on a small instance. In addition, a more detailed analysis of Fig. 4(a) is warranted. We found that OS-TSRO-H did not



(a) Gap in fixed cost of OS-TSRO and VS-TSRO. (b) Gap in outsourcing cost of OS-TSRO and VS-TSRO.

Fig. 5. Gap in fixed cost and outsourcing cost of OS-TSRO and VS-TSRO decisions.

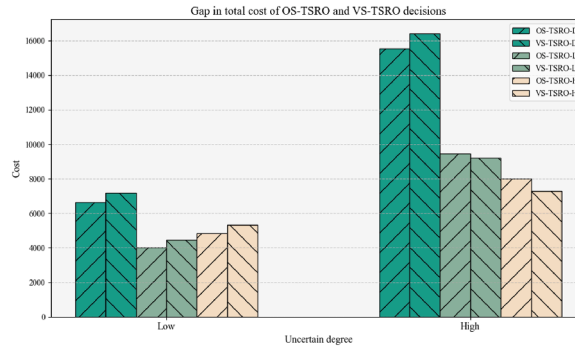


Fig. 6. Gap in total cost of OS-TSRO and VS-TSRO decisions.

demonstrate lower costs at low demand risk. This is attributable to the fact that too many vehicles are put in place, which, when confronted with not so much demand, leads to a decline in the vehicle capacity utilization rate, consequently resulting in asset wastage. Hence, adopting an unconservative or excessively conservative approach may not be optimal for a company. VS-TSRO arrived at analogous conclusions.

Meanwhile, we conducted a comparative analysis of fixed cost, outsourcing cost, and total cost difference between the two collaborative frameworks under the two uncertainty scenarios, as shown in Fig. 5 and Fig. 6. From 5(a), the fixed costs associated with the OS-TSRO decision consistently remain lower, regardless of practical demand risks. This outcome can be attributed to the advantages gained from sharing orders, which enables OS-TSRO to implement optimal commodity divisions that enhance capacity utilization and reduce freight costs. However, this also implies that OS-TSRO's capacity to handle additional freight is diminished. Hence, as demonstrated in Fig. 5(b), the outsourcing cost of OS-TSRO invariably exceeds that of VS-TSRO (with the exception of the deterministic model). Furthermore, to assist freight transportation planners in selecting an appropriate collaborative framework, we present the total transportation costs associated with different collaborative frameworks in the context of actual low and high demand risk. As illustrated in Fig. 6, the order sharing framework exhibits greater economic efficiency under conditions characterized by low demand risks. In contrast, the vehicle sharing framework is recommended for scenarios with higher variability in demand. Additionally, an escalation in unit outsourcing cost will diminish the benefits associated with the order sharing framework. If the freight transportation planners demonstrate a tolerance for risk and favor decision-making within a deterministic context, the order sharing framework is the preferred option. In the next section, we quantitatively analyze the solution structure and provide a detailed explanation of the reasoning behind this conclusion.

5.4. Collaborative performance

In this section, we compare the cost savings rates of two collaborative frameworks under a low degree of uncertainty and analyze how collaboration can be flexible in tackling freight demands. Define $P = (Cost_n - Cost_c) / Cost_n * 100\%$ as the cost savings rate, where $Cost_n$ represents the cost of non-cooperation and $Cost_c$ represents the cost of the order sharing or vehicle sharing framework. The baseline model for the comparison is the non-collaborative case. That is, each carrier fulfills orders independently and cannot lease vehicles or exchange orders. The collaborative carriers utilize disparate budgets to solve the two collaborative models and the non-collaborative model to obtain the decision x using $\hat{d}_k^h = 10$. Subsequently, the high/low risks test demands are used to evaluate the performance of solutions x . All experiments were performed on instance H2K4N6T5. The results are presented in Fig. 7.

As depicted in Fig. 7, the VS-TSRO model does not exhibit any cost savings when the budget is set at 0 or 1. This is because there are fewer customers with fluctuating demand, which negates the necessity for the carrier to lease trucks for transportation purposes. In this case, the solution provided by the VS-TSRO model aligns with the decisions made in a non-collaborative model. As

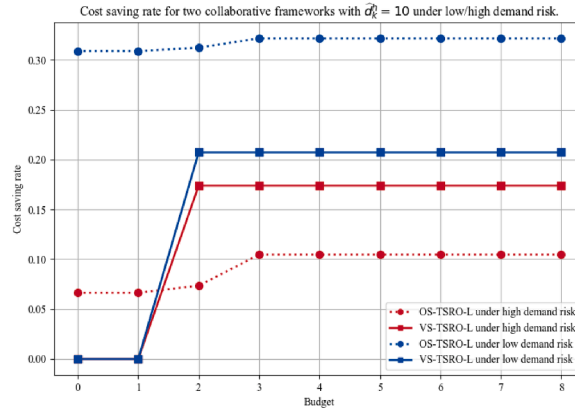


Fig. 7. Cost saving rate for two collaborative frameworks with $\hat{d}_k^h = 10$ under low/high uncertainty degree.

budgets increase, the non-collaborative model fails to yield a more effective solution for managing additional freight with limited trucks. In contrast, the VS-TSRO model demonstrates the capability to optimize transportation costs through the implementation of vehicle sharing, thereby ensuring a more economical approach to delivering customer demands (represented by the solid rectangular line). On the other hand, the OS-TSRO model shows a higher cost-saving rate even as $\Gamma \rightarrow 0$ (illustrated by the dotted circular line). Additionally, the OS-TSRO model exhibits a greater cost-saving rate in scenarios characterized by the low demand risk. This further confirms the conclusions summarized in Section 5.3. It is important to note that both the OS-TSRO and VS-TSRO models demonstrate superior cost-saving rates under low demand risk.

To visualize how collaboration can flexibly handle uncertain demands, we give the service network structure for non-collaboration, OS-TSRO, and VS-TSRO at $\Gamma = 0$ and $\Gamma = 5$, as shown in Fig. 8. The solid and dashed lines represent the movement of vehicles belonging to different carriers between nodes over time. To show the differences in solution structures of the six models quantitatively and intuitively, we define that a cycle in Fig. 8 is a path starting from a node at the beginning of time period 0 and arriving at the same node at the end of time period $|\mathcal{T}| - 1$. First, we observe the impact of uncertainty on the solution structure. As illustrated in Figs. 8(a) and (b), the non-collaborative deterministic solution ($\Gamma = 0$) and the robust solution ($\Gamma = 5$) demonstrate that the latter has as few as two cycles, which is less than the three cycles exhibited by the former. According to Lium et al. (2009) and Wang and Qi (2020), a reduction in the number of cycles results in a greater number of vehicles following the same path, and more consolidation of commodities occurs. Furthermore, we conduct a detailed examination of the freight route of a commodity (the green nodes in the graph represent the departure time and origin, and the red nodes represent the deadline and destination). It has been observed that the deterministic solution provides only two transportation paths for this commodity, whereas the robust solution provides three. This finding demonstrates that the robust solutions can lead to enhanced scheduling flexibility through preparing multiple freight routes. This finding aligns with the results reported by Lium et al. (2009) and Wang and Qi (2020).

Simultaneously, we analyze the solutions of OS-TSRO and VS-TSRO. The solutions of OS-TSRO with $\Gamma = 0$ and $\Gamma = 5$, as shown in Figs. 8(c) and (d), are extremely different from non-collaborative solutions. This is due to the fact that OS-TSRO enables carriers to optimally divide orders and allocate the transportation of goods to the most economically efficient carrier. This phenomenon can be illustrated by the commodity presented in the figure. In the deterministic solutions, this commodity is satisfied independently by one carrier (solid line). In the robust solutions, another carrier is necessary to undertake freight transportation due to fluctuations in customer demand, which fully utilizes the remaining capacity of the vehicle. Hence, as outlined in Section 5.3, this collaborative framework is more advantageous when faced with low actual demand risk. Meanwhile, the solutions of VS-TSRO with $\Gamma = 0$ and $\Gamma = 5$ are as shown in Figs. 8(e) and (f). The solution of VS-TSRO is equivalent to a non-collaborative solution in the absence of vehicle leasing. In instances of increasing demand for more customer orders, non-collaborative carriers are limited by the maximum number of vehicles they can operate, necessitating adjustments to their vehicle routes to address excess demands. However, VS-TSRO has the capability to achieve this through leasing vehicles to each other (in Fig. 8(f), the dotted line from 0 is the vehicle rented by carrier 2). The leased vehicle fulfills an order independently, thereby allowing the vehicle that fulfills the same order under a deterministic solution to be allocated to other customers, which increases network capacity (the purple dashed line defines three distinct commodities). When $\Gamma = 0$, a single vehicle transports two commodities. When $\Gamma = 5$, two vehicles complete the delivery. In consequence, the VS-TSRO advantage is more pronounced in cases of highly volatile demands. The impact of collaborative modes on solution structures is observed by comparing Figs. 8(b), (d), and (f). We can summarize similar findings.

It can thus be concluded that a robust solution has the structural characteristics of using consolidation and path sharing to hedge against uncertainty and provide operational flexibility. In addition, OS-TSRO exhibits a higher cost savings rate in low uncertainty risks by means of optimally dividing orders and selecting carriers; VS-TSRO expands its network capacity by investing in leased vehicles, which shows it has a greater ability to hedge against uncertainty risks.

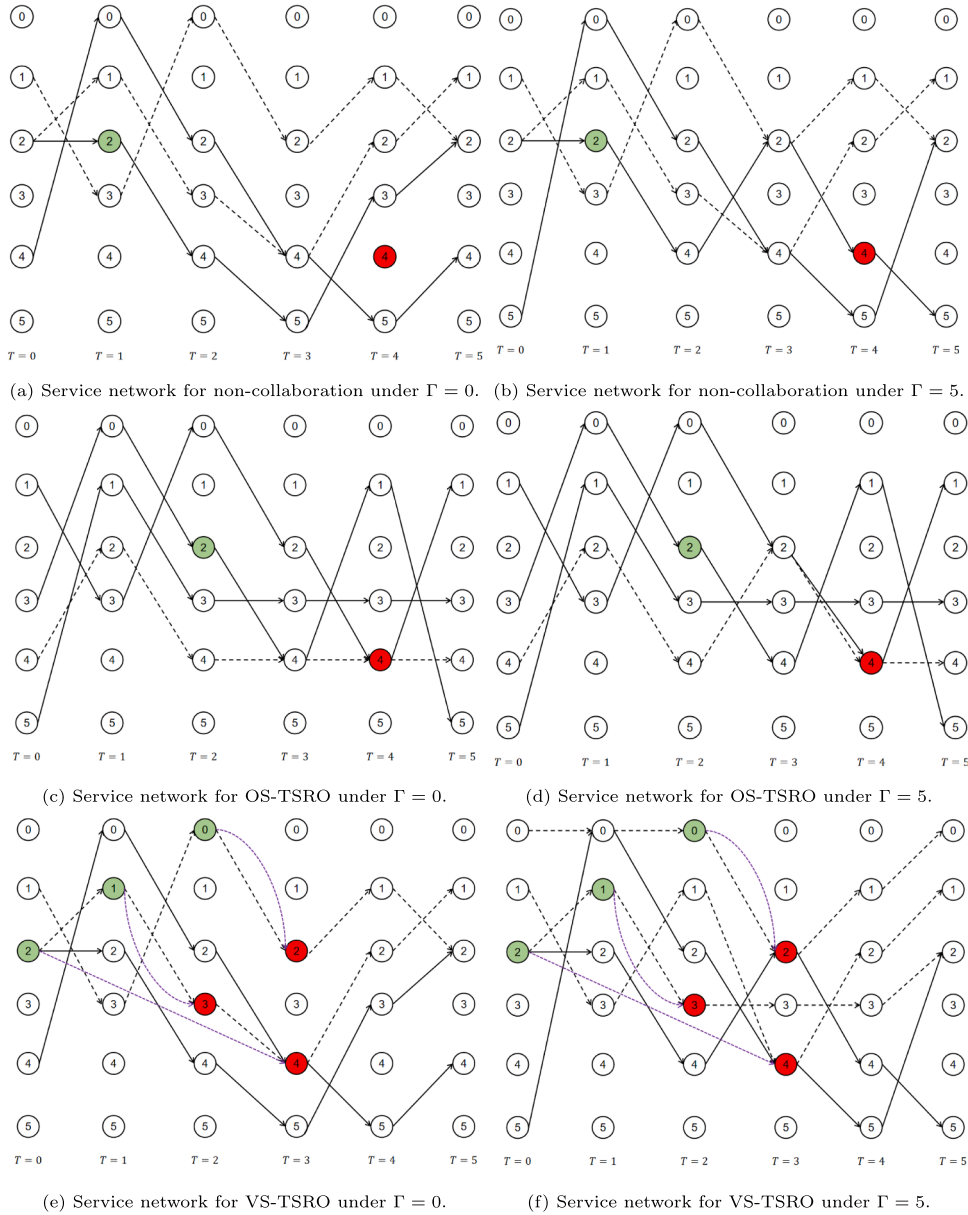


Fig. 8. The service network structure for non-collaboration, OS-TSRO, and VS-TSRO at $\Gamma = 0$ and $\Gamma = 5$.

5.5. Profit-sharing mechanism

In Section 5.4, we discussed the potential cost savings of collaborative models and the uncertain scenarios that they are better suited for in detail. It can be observed that both frameworks reduce the transportation cost to a certain extent. However, in order to enhance their economic efficiency, logistics companies are motivated to engage in horizontal collaboration only when such collaboration is financially advantageous and generates the allocated profit that exceeds the profit of non-collaboration. It is therefore essential to address the rationalization of cost allocation for carriers participating in the collaboration. In this section, the cooperative game core allocation method is utilized to ensure the equitable distribution of cost savings achieved through carrier collaboration.

It is hypothesized that collaborative carriers will form a grand coalition $R = \{0, 1, 2, \dots, |\mathcal{H}| - 1\}$, with the cost savings under the grand coalition R being represented by $P(R)$, where $P(R)$ is a characteristic function value, and it is determined by the cost difference between solutions made under collaboration and non-collaboration. Meanwhile, for $\forall R' \subset R$, we set its characteristic function value, which is denoted $P(R')$. According to the core allocation method of cooperative game theory, we aim to ascertain a set of profit allocation vectors ζ , which ensures the cost savings allocated to any coalition R' are not less than the cost savings that coalition R'

Table 5

The number of instances with cores under two collaboration models.

Instance	OS-TSRO	VS-TSRO
H2K4N6T5	18	18
H3K4N6T5	10	12

would have accrued had it opted out of the grand coalition R . We consider using the *Equal Profit Method* proposed by Frisk et al. (2010). Then the above problem can be formulated into the following optimization model.

$$\min_{\zeta} \sum_{r, r' \in R, r \neq r'} (\zeta_r - \zeta_{r'})^2 \quad (8a)$$

$$\text{s.t. } \zeta(R') \geq P(R'), \quad \forall R' \subset R, \quad (8b)$$

$$\zeta(R) = P(R), \quad (8c)$$

$$\zeta(R') = \sum_{r \in R'} \zeta_r, \quad \forall R' \subset R, \quad (8d)$$

$$\zeta_r \geq 0, \quad \forall r \in R. \quad (8e)$$

where $\zeta_r \in \mathbb{R}^+$ denotes the profit allocated to the player $r \in R$.

Objective 8a is to minimize the discrepancy in profits allocated to individual carriers, thereby ensuring a fairly equitable allocation of cost savings. Constraint 8b represents rationality constraints; Constraint 8c is known as the efficiency constraint, and Constraint 8e limits the range of decision variables. Three freight information tables and six cost matrices were designed for instances H2K4N6T5 and H3K4N6T5. Subsequently, 18 instances from each group were solved in non-collaborative and two collaborative frameworks. The solving results are then input into the aforementioned model. If a solution is found, it can be deduced that a core satisfying equal allocation exists and that the coalition can be stabilized. Conversely, if no feasible solution is found, it can be concluded that the coalition cannot be stabilized under the *Equal Profit Method*. The relevant results are reported in Table 5.

As demonstrated in Table 5, for the VS-TSRO, when $|R| = 2$, all instances under the *Equal Profit Method* possess cores. This is due to the absence of vehicle leasing, rendering VS-TSRO analogous to a non-collaborative model. In this case, the costs of collaboration and non-collaboration are found to be equal. In instances where order volumes exceed the capacity of a given carrier, VS-TSRO avoids the occurrence of expensive outsourcing, which helps stabilize the grand coalition. However, when a new carrier who does not contribute to the overall cost savings also receives an equal profit allocation, this will lead to instability in the grand coalition. As a result, as the number of carriers increases, the number of instances with the core decreases. Similarly, for the OS-TSRO, as the number of carriers increases, the number of instances with the core tends to decrease. This is because we limit the characteristics of the participating collaborative carriers (e.g., capacity, fixed costs). Increasing the number of carriers to a certain point no longer has an effect on determining more cost-effective transportation options by exchanging orders. However, members r may seek to identify coalitions R' where greater cost savings can be achieved. These members are highly likely to break away from the grand coalition, which leads to its destabilization. In general, as shown by the majority of the experimental results, the grand coalition demonstrated instability under the *Equal Profit Method*, in particular when the number of participating carriers increased. Selecting partners judiciously and carefully designing cost-sharing mechanisms may yield additional cost savings and maintain the grand coalition stability; however, this is beyond the scope of this paper.

6. Conclusion

In light of the exponential increase in order volume, it is essential for logistics companies to address the collaborative service network design problem (CSNDP) to mitigate profit losses. This paper investigates the CSNDP within two collaborative frameworks: order sharing and vehicle sharing, particularly under uncertain demands. We have developed the novel two-stage robust optimization model that is based on a budget uncertainty set for each collaborative framework. In light of the NP-hard nature of the problem, we propose the SDS-CCG algorithm, which means the embedding of a linear classifier within the column-and-constraint generation algorithm, aiming to exclude scenarios that are not conducive to convergence in advance. Numerical experiments have been conducted to illustrate the efficiency advantages of the proposed SDS-CCG algorithm, which has demonstrated a significant improvement in solution efficiency for large-scale problems, achieving a 25% reduction in solution time compared to classical column-and-constraint generation methods, while incurring only a minor loss in accuracy. Additionally, we analyze the solution structure characteristics within the two collaborative frameworks and outline the scenarios to which they are applicable. Finally, we examine the grand coalition stability through the lens of core allocation in cooperative game theory.

In the forthcoming research endeavors, we intend to explore several directions: (1) This study requires carriers to form the grand coalition irrespective of their individual preferences. Future research could explore scenarios in which carriers have the autonomy to choose their partners for planning the CSNDP. (2) We embedded a linear classifier into the classical column-and-constraint generation algorithm to identify critical uncertainty scenarios, thereby accelerating algorithm convergence. We hope this will stimulate other researchers to explore similar approaches to further improve the ability to solve practical problems. Future research could focus

on developing more advanced learning methods to identify scenarios that are favorable for convergence. (3) In practical logistics scenarios, each carrier involved in the coalition, motivated by self-interest, typically seeks to maximize its individual profit. This approach contrasts with the framework employed in this study, which entails centralized decision-making succeeded by cost savings allocation. Developing a distributed decision-making method and integrating it into a model constitutes a promising yet challenging research direction. (4) A limitation of this paper is that the network scale utilized is relatively small, which makes it difficult to cover real-world scenarios involving hundreds of nodes. Developing solution methods tailored for large-scale networks and exploring further the conclusions obtained by this paper in large-scale instances constitutes a valuable future research avenue.

CRedit authorship contribution statement

Lingpu Zhang: Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Formal analysis, Conceptualization; **Min Huang:** Writing – review & editing, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization; **Ziteng Wang:** Writing – review & editing, Methodology, Formal analysis; **Xingwei Wang:** Writing – review & editing, Formal analysis; **Shu-Cherng Fang:** Writing – review & editing, Formal analysis.

Data availability

Data will be made available on request.

Declaration of competing interest

None.

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Appendix A. VS-MP and VS-SP

In this section, we give the master problem and subproblem of VS-TSRO. First, we reformulate the internal minimization problem in the second stage of the VS-TSRO as follows.

$$(\text{VS-DRP}(\bar{x}, \bar{V}, \bar{p})) \quad \max_{\alpha, \beta, \gamma} - \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t \setminus \{i\}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} u_h \bar{x}_{ijh}^t \cdot \alpha_{ijh}^t + \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} \left(\beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h} \right) (\bar{d}_k^h + \bar{p}_k^h \hat{d}_k^h) \quad (\text{A1a})$$

$$\text{s.t.} \quad \beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h} \leq \lambda, \quad \forall k \in \mathcal{K}_h, \forall h \in \mathcal{H} \quad (\text{A1b})$$

$$\beta_i^{hkt-} - \beta_j^{hkt} \leq \begin{cases} \alpha_{ijh}^{t-}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}, \quad \forall t \in \mathcal{T}, \forall t^- \in \mathcal{T} \setminus \{\tau_k^h\}, \forall i, j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A1c})$$

$$\beta_i^{hk\tau_k^h} - \beta_j^{hkt} + \gamma_{ijh}^k \leq \begin{cases} \alpha_{ijh}^{\tau_k^h}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}, \quad \forall t \in \mathcal{T}, \forall i, j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A1d})$$

$$\alpha_{ijh}^t \geq 0, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \quad (\text{A1e})$$

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} \bar{p}_k^h \leq \Gamma, \quad (\text{A1f})$$

$$0 \leq \bar{p}_k^h \leq 1, \quad \forall k \in \mathcal{K}_h, \forall h \in \mathcal{H}, \quad (\text{A1g})$$

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} (\bar{d}_k^h + \bar{p}_k^h \hat{d}_k^h) \cdot \rho_k^h + b \geq 0, \quad (\text{A1h})$$

where α, β, γ are the dual variables about constraints 3e, 3g, and 3j, respectively.

In a similar manner, the two-layer maximization problem is transformed into a single-layer maximization problem, and $p_k^h \in \{0, 1\}$.

$$(\text{VS-DRP}(\bar{x}, \bar{V})) \quad \max_{\alpha, \beta, \gamma, p} - \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t \setminus \{i\}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} u_h \bar{x}_{ijh}^t \cdot \alpha_{ijh}^t + \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} \left(\beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h} \right) (\bar{d}_k^h + p_k^h \hat{d}_k^h) \quad (\text{A2a})$$

$$\text{s.t.} \quad \beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h} \leq \lambda, \quad \forall k \in \mathcal{K}_h, \forall h \in \mathcal{H} \quad (\text{A2b})$$

$$\beta_i^{hkt-} - \beta_j^{hkt} \leq \begin{cases} \alpha_{ijh}^{t-}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}, \quad \forall t \in \mathcal{T}, \forall t^- \in \mathcal{T} \setminus \{\tau_k^h\}, \forall i, j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A2c})$$

$$\beta_i^{hk\tau_k^h} - \beta_j^{hkt} + \gamma_{ijh}^k \leq \begin{cases} \alpha_{ijh}^k, & \text{if } i \neq j, \\ 0, & \text{if } i = j \end{cases}, \quad \forall t \in \mathcal{T}, \forall i, j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A2d})$$

$$\alpha_{ijh}^t \geq 0, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \quad (\text{A2e})$$

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} p_k^h \leq \Gamma, \quad (\text{A2f})$$

$$p_k^h \in \{0, 1\}, \quad \forall k \in \mathcal{K}_h, \forall h \in \mathcal{H}, \quad (\text{A2g})$$

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} (\bar{d}_k^h + p_k^h \hat{d}_k^h) \cdot \rho_k^h + b \geq 0, \quad (\text{A2h})$$

Then, we provide a linear reformulation of the model by introducing an auxiliary decision variable w_k^h .

$$w_k^h = \begin{cases} \beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h}, & \text{if } p_k^h = 1 \\ 0, & \text{if } p_k^h = 0 \end{cases}$$

Subsequently, **VS-SP** is defined as follows:

$$\begin{aligned} (\text{VS-SP}) \quad & \max_{\alpha, \beta, \gamma, p} - \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t \setminus \{i\}} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} u_h \bar{x}_{ijh}^t \cdot \alpha_{ijh}^t \\ & + \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} \left(\beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h} \right) \bar{d}_k^h + \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} w_k^h \hat{d}_k^h \end{aligned} \quad (\text{A3a})$$

s.t. [A2b](#) – [A2h](#)

$$w_k^h \leq \beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h}, \quad \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A3b})$$

$$w_k^h + (1 - p_k^h) \cdot M \geq \beta_{o_k^h}^{hk\sigma_k^h} - \beta_{s_k^h}^{hk\tau_k^h}, \quad \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A3c})$$

$$w_k^h \leq M \cdot p_k^h, \quad \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A3d})$$

$$w_k^h \geq 0, \quad \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \quad (\text{A3e})$$

VS-SP defines the subproblem of identifying the worst-case scenario under the given $\mathbf{x}$. Then, the resource variables and constraints determined by the worst-case scenario p_k^{h*} are added to the **VS-MP**.

$$(\text{VS-MP}) \quad \min_{\mathbf{x}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t} \sum_{h \in \mathcal{H}} f_{ijh} \cdot x_{ijh}^t + \sum_{h \in \mathcal{H}} \sum_{h' \in \mathcal{H} \& h' \neq h} \mu \cdot V_{hh'} + \eta \quad (\text{A4a})$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{N}^t} x_{jih}^t = \sum_{j \in \mathcal{N}^t} x_{ijh}^t, \quad \forall i \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall t \in \mathcal{T}, \quad (\text{A4b})$$

$$\sum_{i \in \mathcal{N}^t} \sum_{j \in \mathcal{N}^t} x_{ijh}^t \leq V_h + \sum_{h' \in \mathcal{H} \& h' \neq h} V_{h'h} - \sum_{h' \in \mathcal{H} \& h' \neq h} V_{hh'}, \quad \forall t \in \mathcal{T}, \forall h \in \mathcal{H}, \quad (\text{A4c})$$

$$x_{ijh}^t \in \{0, 1, 2, \dots\}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \quad (\text{A4d})$$

$$V_{hh'} \in \{0, 1, 2, \dots\}, \quad \forall h, h' \in \mathcal{H} \quad h' \neq h, \quad (\text{A4e})$$

$$\eta \geq \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} (Z_k^h)^l, \quad \forall l = 1, \dots, L, \quad (\text{A4f})$$

$$\sum_{k \in \mathcal{K}_h} (y_{ijh}^{kt})^l \leq u_h \cdot x_{ijh}^t, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, i \neq j, \forall h \in \mathcal{H}, \forall l = 1, \dots, L, \quad (\text{A4g})$$

$$\begin{aligned} & \sum_{j \in \mathcal{N}^t} (y_{ijh}^{kt})^l - \sum_{j \in \mathcal{N}^t} (y_{jih}^{kt})^l \\ & = \begin{cases} \bar{d}_k^h + (p_k^{h*})^l \cdot \hat{d}_k^h - (Z_k^h)^l, & \text{if } i = o_k^h, t = \sigma_k^h \\ -\bar{d}_k^h - (p_k^{h*})^l \cdot \hat{d}_k^h + (Z_k^h)^l, & \text{if } i = s_k^h, t = \tau_k^h \\ 0, & \text{otherwise} \end{cases}, \quad \forall i \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, \forall t \in \mathcal{T}, l = 1, \dots, L, \end{aligned} \quad (\text{A4h})$$

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} (p_k^{h*})^l \leq \Gamma, l = 1, \dots, L, \quad (\text{A4i})$$

$$(p_k^{h*})^l \in \{0, 1\}, \quad \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, l = 1, \dots, L, \quad (\text{A4j})$$

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}_h} (\bar{d}_k^h + p_k^{h*} \hat{d}_k^h) \cdot \rho_k^h + b \geq 0, \quad (\text{A4k})$$

$$(y_{ijh}^{krh})^l = 0, \quad \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, l = 1, \dots, L, \quad (\text{A4l})$$

$$(y_{ijh}^{kt})^l \in \mathbb{R}^+, (Z_k^h)^l \in \mathbb{R}^+, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^t, \forall j \in \mathcal{N}^t, \forall h \in \mathcal{H}, \forall k \in \mathcal{K}_h, l = 1, \dots, L, \quad (\text{A4m})$$

where $(p_k^{h*})^l$ is the worst-case scenario identified by **VS-SP** for the l th iteration. **A4f** - **A4m** are resource constraints.

Solve **VS-SP** and **VS-MP** iteratively until convergence of the upper and lower bounds is achieved, as outlined in the algorithmic framework provided in Section 4.3.

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