

## A COOPERATIVE GAME WITH ENVY

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(Communicated by Kok Lay Teo)

**ABSTRACT.** This paper proposes an envy-incorporated cooperative game model and investigates the envy effects on players' coalition-forming and payoff-allocating decisions. A player's utility depends on her/his own payoff and the envy toward other players, inside and outside the same coalition, with higher payoffs. Envy core is defined to characterize stable coalition structures and payoff allocations of this new game. Conditions for the envy core to be nonempty are provided. The relative significance of in-coalition envy and out-coalition envy is shown to be a key factor to the form of the envy core. Application to a simple game shows that envy may significantly change players' decisions.

**1. Introduction.** Envy is a common emotion that “occurs when a person lacks another's superior quality, achievement, or possession and either desires it or wishes that the other lacked it” [28]. Envy prevails in social and organization life [11]. In a social choice environment, envy is one's unhappy feeling if s/he is in a disadvantageous position when comparing her/his payoff with others'. For example, Frank [16] observes that people in workplace always compare their rewards with coworkers. Workers' utilities may go down when others' wages go up [31]. In this sense, envy may harm an employee's job satisfaction [40]. To the best of our knowledge, research about the envy effects on social behaviors is predominantly conceptual while its theoretic counterpart is highly necessary.

Theory of cooperative games has been extensively developed and successfully applied in economics, sociology, political science, psychology, and biology. In a cooperative game, players are assumed to be perfectly and equally rational. Possible binding contracts may induce players to form coalitions and achieve higher payoff

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2010 *Mathematics Subject Classification.* Primary: 91A12, 91A30.

*Key words and phrases.* Cooperative game, core, envy, coalition, behavior.

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than stand-alone. Solution concepts such as core [17] and Shapley value [35] are often used to characterize stable outcomes of a cooperative game. However, the rationality assumption may not fully capture the reality. For instance, studies on the majority rule games [13] show that the classic cooperative game theory may fail to take into account players' behavioral preferences on the payoff allocations. Such preference of others' payoffs over one's own may lead to envy [18].

This study incorporates envy into a classic cooperative game to investigate the effect of envy on players' coalition formation and payoff allocation. We focus on super-additive cooperative games, which means that a coalition always yields a higher payoff than all its subcoalitions in total. Payoff of each coalition can be allocated among its members, but inter-coalition payoff transfer is prohibited. We assume that the utility of each player is a linear combination of her/his own payoff and the utility loss caused by envy when other players get higher payoffs. A player may have different levels of envy toward the players in the coalition s/he belongs to (in-coalition envy) and the players out of the coalition s/he belongs to (out-coalition envy). We associate different factors for in-coalition envy and out-coalition envy in the utility function. Players make decisions of coalition formation and payoff allocation based on their utilities instead of payoffs. This new model of cooperative game with envy generalizes the classic cooperative game. When the in-coalition envy and out-coalition envy are both zero, the utility of a player is equal to the payoff s/he is allocated. However, the new cooperative game with envy is no longer a transferable utility game.

We propose solution concepts of the cooperative game with envy. A coalition structure refers to a partition of all players. A solution consists of a coalition structure and a payoff allocation of each of its coalitions. A solution is stable when no players can form a new coalition, with a payoff allocation, such that each member of this new coalition can guarantee a higher utility under all circumstances. We define the envy core as the set of all stable solutions. Envy core extends the core concept in classic super-additive cooperative games, where the grand coalition (coalition of all players) is always preferred and only payoff allocation is of interest.

We provide conditions under which the envy core is nonempty. When the in-coalition envy is no greater than the out-coalition envy, we find that the grand coalition is stable as long as the envy core is nonempty. In this case, as the out-coalition envy increases, payoff allocations in the envy core may show larger variation. When the in-coalition envy is greater than the out-coalition envy, the envy core may contain coalitions other than the grand coalition. We also study the extreme cases when in-coalition envy or out-coalition envy is infinity and find explicit forms of the envy core when it is nonempty. Moreover, we show that the envy core is nonempty when there exists one or multiple players who can form a coalition with higher per-capita payoffs.

Applying our new model and analysis on a simple game, we find that envy may significantly change the roles and decisions of players in a voting practice. Compared with known results of the classic simple game, non-veto players may have more bargaining power against the veto players due to in-coalition envy.

The rest of the paper is arranged as follows. Section 2 reviews envy-related literature in behavioral game theory and social identity theory. Section 3 proposes a cooperative game with envy and introduces some new solution concepts. Section 4 studies the properties of the envy core. Section 5 applies this study to a simple game. Section 6 concludes the paper. All proofs are included in the appendix.

**2. Literature review.** In this section, we review literature that studies envy in behavioral game theory and social choices.

**2.1. Envy in behavioral game theory.** In the behavioral game theory, envy has been primarily investigated for ultimatum bargaining games. In such a game, a proposer makes an offer to a responder suggesting how to divide a certain amount of money. If the responder accepts the offer, then the money is allocated accordingly. Otherwise, both players receive nothing. The best (rational) strategy of the proposer is to offer a small amount to the responder while the best (rational) decision of the responder is to accept no matter how small a positive amount s/he is offered. In experimental studies, however, the means of most offers stabilize at the level of 40 – 50% of the total amount of money. Offers that are too low are often rejected [3, 7]. Similar phenomena are observed in the multi-period ultimatum game [5, 22]. These games are robust in variation of the amount of money to be divided [8, 32] and cultures [19, 32]. Kirchsteiger [22] considers envy in the bargaining model and shows that envy explains most experimental “anomalies” satisfyingly. Moreover, in the alternating-bargaining game, envy has been shown to promote close divisions [23].

Okada and Riedl [26] show that envy affects coalition formation in a one-proposer-two-responder ultimatum game. The proposer can either form a two-player coalition with one of the responders to divide a smaller amount of money, or form a three-player coalition (the grand coalition) with the two responders to divide a bigger amount of money. In their experiments, a majority of proposers choose the inefficient two-player coalition forced by their anticipation of the responders’ envy. Therefore, envy may lead to social inefficiency and social exclusion.

**2.2. Envy and social identity.** The theory of group identity was introduced to economics by Akerlof and Kranton [1]. The so-called other-other allocation game, where a player is asked to allocate a certain amount of money to others, is used to study the interaction of group identity and envy. Experimental data from the other-other allocation game consistently indicates in-group favoritism and out-group discrimination [39]. That is, individuals in the same group as the money-allocating player receive significantly bigger allocation than individuals in other groups. Chen and Li [10] find that participants exhibit significantly less envy when an in-group member gets a higher payoff than the case when an out-group member does. Additionally, the relation between group identity and envy remains strong even when the no-conflict setting between self interest and group interest is removed. Seta and Seta [34] find that individuals at inferior positions within a group prefer to join another group because inferior status within a group leads to envy [28].

**3. Game model and solution concepts.** In this section, we propose a cooperative game with envy by incorporating each player’s envy toward other players’ payoff into her/his utility. Solution concepts of this new game model are provided.

In this paper, we use  $N = \{1, 2, \dots, n\}$  to denote the set of a finite number of  $n$  players, indexed by  $i = 1, 2, \dots, n$ . We assume that  $n > 1$  throughout the paper. Moreover, we assume that the initial wealth of each player is zero following literature [38]. Let  $2^N$  denote the set of all subsets of  $N$ . A nonempty subset  $S \subseteq N$ , with cardinality  $|S|$ , is called a coalition.  $N$  is called the grand coalition.

We define a coalition structure  $\mathcal{P}$  as a partition over  $N$ , i.e.,  $\mathcal{P} = \{S_1, S_2, \dots, S_K\}$  such that  $S_k \neq \emptyset$  for  $k = 1, \dots, K$ ,  $\cup_{k=1}^K S_k = N$ , and  $S_{k_1} \cap S_{k_2} = \emptyset$  for

$k_1, k_2 = 1, \dots, K$  and  $k_1 \neq k_2$ . Denote the grand coalition structure  $\{N\}$  by  $\mathcal{P}_N$ . Given a coalition structure  $\mathcal{P}$ , the coalition that player  $i$  belongs to is denoted by  $S^{\mathcal{P}}(i)$ . Note that  $S^{\mathcal{P}}(i)$  is unique by the nature of partition.

Let  $\mathbb{R}^m$  denote the  $m$ -dimensional real space. A function  $v : 2^N \rightarrow \mathbb{R}^1$  is called super-additive if  $v(S_1 \cup S_2) \geq v(S_1) + v(S_2)$  for any two disjoint  $S_1, S_2 \subseteq N$ . The function  $v$  is used to characterize the payoff of any coalition  $S \subseteq N$  that is available to be allocated among its members. We assume that the payoff function  $v$  is super-additive in the remaining of the paper. Moreover, following classical literature [2, 29], we assume  $v(\emptyset) = 0$ . Consequently,  $v(S) \geq 0$  for all  $S \subseteq N$ .

We retain the following two assumptions on payoff allocation in classic cooperative games:

- (A1) (Binding Contracts) Players in a coalition can make agreement of the allocation of the coalition's payoff.
- (A2) (Side Payments) Intra-coalitional payoff transfer is allowed and inter-coalitional payoff transfer is prohibited.

Note that the assumption (A2) only permits the transfer of monetary payoff to be within each coalition [29]. For any nonempty coalition  $S = \{i_1, i_2, \dots, i_{|S|}\} \subseteq N$ , we denote  $\mathbf{x}^S = (x_{i_1}^S, \dots, x_{i_{|S|}}^S), i_j \in S, j = 1, \dots, |S|$ , as the payoff allocation vector of players in  $S$ . We say  $\mathbf{x}^S$  is feasible for  $S$  if  $\mathbf{x}^S \geq 0$  and  $\sum_{i \in S} x_i^S = v(S)$ . That is, each player in  $S$  receives a nonnegative payoff and  $v(S)$  is fully allocated among players in  $S$ . By restricting a feasible payoff allocation to be nonnegative, we eliminate the cases when a player has to pay other players in order to become a member of a coalition, given her/his initial wealth being zero, so that we can focus on the allocation of  $v(S)$ . Notice that the individual rationality assumption of  $x_i^S \geq v(i)$  in classic cooperative games is no longer assumed in this paper because, as we will see, a player may sacrifice part of her/his stand-alone payoff in order to be a member of a coalition due to a large out-coalition envy. Let  $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$  denote a payoff allocation vector of all players in  $N$ . We say  $\mathbf{x}$  is feasible for coalition structure  $\mathcal{P}$  if  $\mathbf{x} \geq 0$  and  $\sum_{i \in S} x_i = v(S)$  for all  $S \in \mathcal{P}$ .

In classic cooperative games, a player's utility is equal to her/his own payoff. In this research, we assume that envy may affect a player's utility.

- (A3) (Envy) A player dislikes that the payoff allocated to any other player is higher than her/his allocated payoff.

To incorporate envy into the utility function of a player, we adopt a variation of the model in Fehr and Schmidt [14]. If players  $i$  and  $j$  are in the same coalition, then the envy of  $i$  toward  $j$  is characterized by

$$-\alpha \frac{1}{|S^{\mathcal{P}}(i)| - 1} \max(x_j - x_i, 0),$$

with  $\alpha \geq 0$ . Here the parameter  $\alpha \geq 0$  is given as the in-coalition envy.  $|S^{\mathcal{P}}(i)| - 1$  is used as a normalizing factor. If players  $i$  and  $j$  are in different coalitions, then the envy of  $i$  toward  $j$  is characterized by

$$-\beta \frac{1}{n - |S^{\mathcal{P}}(i)|} \max(x_j - x_i, 0),$$

with  $\beta \geq 0$ . The parameter  $\beta \geq 0$  is given as the out-coalition envy.  $n - |S^{\mathcal{P}}(i)|$  is for normalization. In this way, given  $\alpha, \beta \geq 0$ , we may define the utility of player  $i$  as a function of the coalition structure  $\mathcal{P}$  and its feasible payoff allocation  $\mathbf{x}$  as

following:

$$\mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta) = \begin{cases} x_i - \beta \frac{1}{n-1} \sum_{j \neq i} \max(x_j - x_i, 0), & \text{if } |S^{\mathcal{P}}(i)| = 1, \\ x_i - \alpha \frac{1}{|S^{\mathcal{P}}(i)| - 1} \sum_{j \in S^{\mathcal{P}}(i), j \neq i} \max(x_j - x_i, 0) \\ \quad - \beta \frac{1}{n - |S^{\mathcal{P}}(i)|} \sum_{j \notin S^{\mathcal{P}}(i)} \max(x_j - x_i, 0), & \text{if } 1 < |S^{\mathcal{P}}(i)| < n, \\ x_i - \alpha \frac{1}{n-1} \sum_{j \neq i} \max(x_j - x_i, 0), & \text{if } |S^{\mathcal{P}}(i)| = n. \end{cases}$$

In this paper, we define  $+\infty \cdot 0 = 0$ . Then  $\mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta)$  applies to the cases when  $\alpha = +\infty$  or  $\beta = +\infty$ . Let  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_n)$ . Note that when  $\alpha = \beta = 0$ , player  $i$ 's utility  $\mu_i(\mathcal{P}, \mathbf{x}; 0, 0)$  becomes her/his payoff  $x_i$ .

Now we propose a cooperative game with envy and some solution concepts.

**Definition 3.1.** A cooperative game with envy is a triple  $(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ .

Note that when  $\alpha > 0$  or  $\beta > 0$ , a cooperative game with envy is a non-transferable utility game.

**Definition 3.2.** A solution to a cooperative game with envy  $(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$  is a pair of a coalition structure  $\mathcal{P}$  and its feasible payoff allocation vector  $\mathbf{x}$ , i.e.,  $(\mathcal{P}, \mathbf{x})$ .

A solution represents a possible outcome of coalition formation and payoff allocation in a cooperative game with envy. We study the stableness of a solution by extending the ideas of classic stable solution [17, 35] and strong egalitarian solution [12]. Intuitively speaking, if no players can guarantee higher utilities by forming a new coalition or agreeing on a new payoff allocation, then the current solution, consisting of the coalition structure and the payoff allocation, is stable. When a player considers joining some other players to form a new coalition with a new payoff allocation, s/he is uncertain about the outcome of the remaining players outside of the potential new coalition. This uncertainty affects the envy between players of different coalitions. In this study, we employ the commonly used worst-case analysis in decision theory and game theory. Note that for any player  $i$  in a coalition  $S \neq N$  with a feasible payoff allocation  $\mathbf{x}^S$ , in any solution  $(\mathcal{P}, \mathbf{x})$  such that  $S \in \mathcal{P}$  and  $x_i = x_i^S$ , the aggregated envy of player  $i$  toward all players outside  $S$  satisfies the following inequalities:

$$\sum_{j \notin S} \max(x_j - x_i^S, 0) \leq \max\left(\sum_{j \notin S} x_j - x_i^S, 0\right) \leq \max(v(N \setminus S) - x_i^S, 0),$$

where both equalities hold when players in  $N \setminus S$  form a single coalition and one player gets allocated all payoff  $v(N \setminus S)$ . Therefore, for any given coalition  $S$  with a feasible payoff allocation  $\mathbf{x}^S$ , the lowest utility that a player  $i \in S$  may have, denoted by  $\underline{\mu}_i(S, \mathbf{x}^S; \alpha, \beta)$ , can be expressed as

$$\underline{\mu}_i(S, \mathbf{x}^S; \alpha, \beta) = \begin{cases} x_i^S - \beta \frac{1}{n-1} \max(v(N \setminus S) - x_i^S, 0), & \text{if } |S| = 1, \\ x_i^S - \alpha \frac{1}{|S| - 1} \sum_{j \in S, j \neq i} \max(x_j^S - x_i^S, 0) \\ \quad - \beta \frac{1}{n - |S|} \max(v(N \setminus S) - x_i^S, 0), & \text{if } 1 < |S| < n, \\ x_i^S - \alpha \frac{1}{n-1} \sum_{j \neq i} \max(x_j^S - x_i^S, 0), & \text{if } |S| = n. \end{cases}$$

A formal definition of stable solutions is given below.

**Definition 3.3.** In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , a solution  $(\mathcal{P}, \mathbf{x})$  is stable if there exists no coalition  $S \subseteq N$  with feasible payoff allocation  $\mathbf{x}^S$  such that  $\underline{\mu}_i(S, \mathbf{x}^S; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta)$  holds for all  $i \in S$ .

The coalition structure  $\mathcal{P}$  of a stable solution  $(\mathcal{P}, \mathbf{x})$  is called a stable coalition structure. We now extend the concept of core in classic cooperative games and propose an envy core.

**Definition 3.4.** The envy core of a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$  is defined as the set of all stable solutions and denoted by  $\mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ .

When  $\alpha = \beta = 0$ , the envy core becomes the core of a classic cooperative game. Actually, if  $\mathbf{x}$  is in the core of the classic and super-additive cooperative game  $(N, v)$ , then  $(\mathcal{P}_N, \mathbf{x})$  is a stable solution to  $(N, v, \mu(\cdot, \cdot; 0, 0))$ . If  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \mu(\cdot, \cdot; 0, 0))$ , then, as we will show in the next section,  $\mathbf{x}$  is in the core of  $(N, v)$ . Thus the envy core is a generalization of the classic core [27].

**4. Properties of the envy core.** In this section, we investigate the nonemptiness and characterization of the envy core. The in-coalition envy, out-coalition envy and some key player(s) will play important roles in our analysis.

In a classic super-additive cooperative game, the grand coalition is preferred because it generates the largest total payoff. In this case, the concept of core represents stable allocations of the total payoff  $v(N)$  of the grand coalition. In a cooperative game with envy, however, we find that the grand coalition structure  $\mathcal{P}_N$  may not be stable.

**Example 1.** Let  $N = \{1, 2, 3\}$ ,  $v(1) = v(2) = 0$ ,  $v(3) = 2$ ,  $v(\{1, 2\}) = 1$ ,  $v(\{1, 3\}) = v(\{2, 3\}) = 2$ , and  $v(\{1, 2, 3\}) = 3$ .  $\alpha = 1$  and  $\beta = 0$ .

In this cooperative game with envy,  $\mathcal{P} = \{\{1, 2\}, \{3\}\}$  is the only stable coalition structure, with feasible payoff allocation  $\mathbf{x}$ , satisfying

$$\frac{1}{3} \leq x_1 \leq \frac{2}{3}, x_2 = 1 - x_1, x_3 = 2.$$

Notice that  $\alpha > \beta$  in Example 1. In this case, players 1 and 2 prefer not to participate in the grand coalition in order to avoid the relatively high in-coalition envy toward the player 3. Instead, players 1 and 2 form a coalition while the player 3 is left with a single coalition.

When  $\alpha \leq \beta$ , we have the next theorem that characterizes the envy core, if it is nonempty.

**Theorem 4.1.** In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , if  $0 \leq \alpha \leq \beta < +\infty$  and the envy core is nonempty, then for any stable solution  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , we have

$$\sum_{i \in N} x_i = v(N), \quad (1)$$

$$\mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta) = \mu_i(\mathcal{P}_N, \mathbf{x}; \alpha, \beta), \forall i \in N. \quad (2)$$

In particular,  $\mathcal{P}_N$  is a stable coalition structure.

Theorem 4.1 implies that when the in-coalition envy is no greater than the out-coalition envy, the grand coalition structure is stable as long as the envy core is nonempty. Moreover, in terms of total payoff and individual utilities, any stable solution  $(\mathcal{P}, \mathbf{x})$  is as good as  $(\mathcal{P}_N, \mathbf{x})$ . This could help us understand a cooperative

game with envy from the viewpoint of a general cooperative game without transferable payoff. Using Theorem 4.1, we could study conditions for the nonemptiness of the envy core from the perspective of the payoff functions.

**Example 2.** Suppose  $\alpha = 0$  and  $\beta = 2$  in Example 1. Then the grand coalition structure  $\mathcal{P}_N = \{\{1, 2, 3\}\}$  is the unique stable coalition structure, with feasible payoff allocation  $\mathbf{x}$ , satisfying

$$x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1, x_1 + x_2 + x_3 = 3.$$

The next lemma shows how the envy core changes as  $\beta$  increases when  $\alpha \leq \beta$ .

**Lemma 4.2.** *For two cooperative games with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta_1))$  and  $(N, v, \mu(\cdot, \cdot; \alpha, \beta_2))$ , if  $0 \leq \alpha \leq \beta_1 \leq \beta_2 < +\infty$ , then  $\mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta_1)) \subseteq \mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta_2))$ .*

As Theorem 4.1 indicates, we may consider only the grand coalition structure. Lemma 4.2 implies that, when  $0 \leq \alpha \leq \beta$  and the out-coalition envy  $\beta$  increases, players tend to stay in the grand coalition to avoid a higher out-coalition envy. On the other hand, players may accept in-coalition envy more than out-coalition envy. Consequently, the set of stable payoff allocations of the grand coalition may enlarge. This result complements the result of Montero [25] which shows that inequity aversion bargain among players may increase payoff inequity.

**Example 3.** Suppose  $\alpha = 0$  in Example 1 and let  $\beta$  vary. The envy core can be expressed as

$$\begin{aligned} \mathcal{C}(N, v, \mu(\cdot, \cdot; 0, \beta)) = & \left\{ (\mathcal{P}, \mathbf{x}) \mid \mathcal{P} = \{\{1, 2, 3\}\}, x_1 \geq 0, x_2 \geq 0, \right. \\ & \left. \max(1 - 3\beta, 0) \leq x_1 + x_2 \leq 1, x_1 + x_2 + x_3 = 3 \right\}. \end{aligned}$$

Obviously,  $\mathcal{C}(N, v, \mu(\cdot, \cdot; 0, \beta_1)) \subseteq \mathcal{C}(N, v, \mu(\cdot, \cdot; 0, \beta_2))$  for  $0 \leq \beta_1 \leq \beta_2$ . As  $\beta$  increases, it becomes more tolerable for players 1 and 2 to see that the player 3 gets a higher payoff.

To further study the nonemptiness of the envy core, we review the concept of balanced map and Bondareva-Shapley theorem in classic cooperative game theory. We associate any given nonempty set  $S \subseteq N$  with a vector  $\mathbf{e}^S \in \mathbb{R}^n$  whose entries are defined by

$$\mathbf{e}_i^S = \begin{cases} 1, & \text{if } i \in S, \\ 0, & \text{otherwise.} \end{cases}$$

Accordingly,  $\mathbf{e}^N = (1, \dots, 1) \in \mathbb{R}^n$ .

**Definition 4.3.** [29] A function  $\lambda : 2^N \setminus \{\emptyset\} \rightarrow \mathbb{R}_+^1$  is called a balanced map if

$$\sum_{S \in 2^N \setminus \{\emptyset\}} \lambda(S) \mathbf{e}^S = \mathbf{e}^N.$$

A balanced map provides a “fractional partition” over the set of players [29]. By using balanced map, the Bondareva-Shapley theorem [6, 36] provides a necessary and sufficient condition for the core to be nonempty in a classic cooperative game ( $\alpha = \beta = 0$ ). In this paper, we rephrase it in a context of cooperative games with envy.

**Theorem 4.4.** (*Bondareva-Shapley*) In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , if  $\alpha = \beta = 0$ , then the envy core  $\mathcal{C}(N, v, \mu(\cdot, \cdot; 0, 0))$  is nonempty if and only if

$$\sum_{\emptyset \neq S \subseteq N} \lambda(S) v(S) \leq v(N)$$

holds for all balanced map  $\lambda$ .

As a direct consequence of Lemma 4.2 and Theorem 4.4, we can provide a sufficient condition for the nonemptiness of envy core when  $\alpha = 0$  and  $0 \leq \beta < +\infty$ .

**Theorem 4.5.** In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; 0, \beta))$ , if

$$\sum_{\emptyset \neq S \subseteq N} \lambda(S) v(S) \leq v(N) \quad (3)$$

holds for all balanced map  $\lambda$ , then  $\mathcal{C}(N, v, \mu(\cdot, \cdot; 0, \beta)) \neq \emptyset$ , for  $0 \leq \beta < +\infty$ , and  $\mathcal{P}_N$  is a stable coalition structure of the game.

When  $\alpha = 0$  and  $\beta = +\infty$ , a player can not tolerate that any player in a different coalition has a higher payoff than hers/his. In this case, a coalition of players  $S$  that satisfies  $\frac{v(S)}{|S|} < v(N \setminus S)$  have no intention to leave the grand coalition because, otherwise, at least one player in  $S$  may have the lowest utility of  $-\infty$ . Therefore, we have the next theorem that gives a weaker sufficient condition than that in Theorem 4.5 for the envy core to be nonempty.

**Theorem 4.6.** In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; 0, +\infty))$ , if

$$\sum_{\substack{S: \emptyset \neq S \subseteq N, \\ \frac{v(S)}{|S|} \geq v(N \setminus S)}} \lambda(S) v(S) \leq v(N) \quad (4)$$

holds for all balanced map  $\lambda$ , then  $\mathcal{C}(N, v, \mu(\cdot, \cdot; 0, +\infty)) \neq \emptyset$ .

For the other two extreme cases,  $\alpha = +\infty, \beta = 0$  and  $\alpha = \beta = +\infty$ , we show that the respective envy core is nonempty.

**Theorem 4.7.** In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; +\infty, 0))$ , the envy core  $\mathcal{C}(N, v, \mu(\cdot, \cdot; +\infty, 0))$  is nonempty.

**Theorem 4.8.** In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; +\infty, +\infty))$ , the envy core  $\mathcal{C}(N, v, \mu(\cdot, \cdot; +\infty, +\infty))$  is nonempty.

**Example 4.** In Example 1, if  $\alpha = +\infty$  and  $\beta = 0$ , or  $\alpha = \beta = +\infty$ , we can verify that

$$(\mathcal{P}, \mathbf{x}) = \left( \{\{1, 2\}, \{3\}\}, (0.5, 0.5, 2) \right)$$

is a stable solution.

Theorems 4.7, 4.8 and Example 4 indicate that when  $\alpha = +\infty$ , as a social preference, a player desires in-coalition equity, which coincides with the central idea of the strong egalitarian solution [12, 24].

In general, when  $\alpha > 0$ , determining whether the envy core is nonempty can become very complicated. Nevertheless, in some cases when some players play a more important role in terms of payoff compared to other players, we can derive sufficient conditions for the envy core to be nonempty.

**Theorem 4.9.** *In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , if there is a player  $i_0$  such that  $v(\{i_0\}) > \frac{v(S)}{|S|}, \forall S \subseteq (N \setminus \{i_0\}), S \neq \emptyset$ , then the envy core  $\mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta))$  is nonempty when  $\beta$  is sufficiently large.*

In Theorem 4.9, player  $i_0$  is special in the sense that at least one player in  $S$  suffers envy toward  $i_0$  for any  $\emptyset \neq S \subseteq (N \setminus \{i_0\})$ . When the out-coalition envy  $\beta$  is large enough, players tend to stay in the same coalition with  $i_0$ . Consequently, the grand coalition structure  $\mathcal{P}_N$  is stable. Moreover, player  $i_0$  may take advantage of this case and grab a large part of the payoff of the grand coalition. For a similar reasoning, we have the next result.

**Theorem 4.10.** *In a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , if there is a set of players  $S_0 \subseteq N, |S_0| > 1$ , such that  $\frac{v(S_0)}{|S_0|} > \frac{v(S \cup (N \setminus S_0))}{|S|}, \forall S \subsetneq S_0, S \neq \emptyset$ , then the envy core  $\mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta))$  is nonempty when  $\beta$  is sufficiently large.*

Similar to Kahan and Rapaport [20], Theorem 4.10 studies the case in which some players are stronger while others are relatively weaker. When  $\beta$  is large enough, the strong players ( $S_0$ ) may be allocated most of the payoff of the grand coalition, and the other weak players ( $N \setminus S_0$ ) still prefer to stay with the strong players. Consequently, the grand coalition structure  $\mathcal{P}_N$  is stable. Additionally, we point out that the condition in Theorem 4.10 are satisfied by the majority voting game [29].

Note that the meaning of stability of a solution and the analysis of the envy core in this paper are based on the worse-case scenario by which a player evaluates the uncertainty of payoff allocation among the out-coalition players. This rule indicates that a player assumes any feasible out-coalition allocation is possible to be realized. When other principles for describing the behavior of the out-coalition players are employed [41], the results of stable coalition structures and envy core may change. Research along this direction can bring new insights regarding the effects of envy on coalition formation and payoff allocation.

In the remaining of this section, we provide an understanding of a cooperative game with envy from the perspective of non-transferable utility games (NTU games, [27]). Given a cooperative game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , we can define the set of consequences as

$$\Omega = \{(\mathcal{P}, \mathbf{x}) \mid \mathcal{P} \text{ is a coalition structure and } \mathbf{x} \text{ is feasible for } \mathcal{P}\}.$$

For each nonempty coalition  $S \neq N$ , define an associated subset of  $\Omega$  as

$$V(S) = \{(\{S, N \setminus S\}, (\mathbf{x}^S, \mathbf{x}^{N \setminus S})) \in \Omega \mid \mathbf{x}^{N \setminus S} \text{ has only one non-zero coordinate which equals to } v(N \setminus S)\}.$$

Let  $V(N) = \Omega$ . For each player  $i \in N$ , the preference relation on  $\Omega$  is generated by the utility function  $\mu_i(\cdot, \cdot; \alpha, \beta)$ . Consider the NTU game  $(N, \Omega, V, \mu(\cdot, \cdot; \alpha, \beta))$ . The core of  $(N, \Omega, V, \mu(\cdot, \cdot; \alpha, \beta))$  ([27]) becomes

$$\begin{aligned} \mathcal{C}(N, \Omega, V, \mu(\cdot, \cdot; \alpha, \beta)) = \{(\mathcal{P}, \mathbf{x}) \in \Omega \mid \forall S \subseteq N, \nexists (\bar{\mathcal{P}}, \bar{\mathbf{x}}) \in V(S) \\ \text{such that } \mu_i(\bar{\mathcal{P}}, \bar{\mathbf{x}}; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta) \text{ for } i \in S\}. \end{aligned}$$

With the above settings, the envy core of  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$  can be expressed as following:

$$\begin{aligned} \mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta)) = \{(\mathcal{P}, \mathbf{x}) \in \Omega \mid \forall S \neq N, \nexists (\bar{\mathcal{P}}, \bar{\mathbf{x}}) \in V(S) \\ \text{such that } \mu_i(\bar{\mathcal{P}}, \bar{\mathbf{x}}; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta) \text{ for } i \in S\} \end{aligned}$$

$$\cap \{(\mathcal{P}, \mathbf{x}) \in \Omega \mid \nexists (\{N\}, \mathbf{x}^N) \in \Omega$$

$$\text{such that } \mu_i(\{N\}, \mathbf{x}^N; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta) \text{ for } i \in N\}.$$

It is easy to verify that  $\mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) \subseteq \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ , i.e., the core is contained in the envy core. Moreover, consider the Pareto frontier of the envy core as following:

$$PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))) = \{(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) \mid \nexists (\bar{\mathcal{P}}, \bar{\mathbf{x}}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$$

$$\text{such that } \mu_i(\bar{\mathcal{P}}, \bar{\mathbf{x}}; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta) \text{ for } i \in N\}.$$

We have

**Theorem 4.11.** *Let  $(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$  be a cooperative game with envy and  $(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$  be the corresponding non-transferable utility game. Then  $\mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) = PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)))$ .*

Theorem 4.11 means that the core of the NTU game is the Pareto frontier of the envy core. The next theorem points out that the core of the NTU game can be equal to the envy core when the in-coalition envy is no greater than the out-coalition envy.

**Theorem 4.12.** *Let  $(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$  be a cooperative game with envy and  $(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$  be the corresponding non-transferable utility game. If  $0 \leq \alpha \leq \beta < +\infty$ , then  $\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) = \mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ .*

The proof is simple by noting Theorem 4.1.

When  $\alpha > \beta$ , the example below shows that the core of the NTU game and the envy core may not be equal.

**Example 5.** Let  $N = \{1, 2, 3, 4, 5\}$ .  $v(\{i\}) = 0$  for  $i \in N$ ;  $v(\{1, 2\}) = 18.002$ ,  $v(\{4, 5\}) = 12$ ,  $v(\{i, j\}) = 0$  for other  $\{i, j\} \subseteq N$  with  $i \neq j$ ;  $v(\{1, 2, 3\}) = 26$ ,  $v(\{1, 2, 4\}) = v(\{1, 2, 5\}) = 18.002$ ,  $v(\{1, 4, 5\}) = v(\{2, 4, 5\}) = 12$ ,  $v(\{3, 4, 5\}) = \frac{58}{3}$ ,  $v(\{i, j, k\}) = 0$  for other  $\{i, j, k\} \subseteq N$  with  $i \neq j$ ,  $i \neq k$ ,  $j \neq k$ ;  $v(\{1, 2, 3, 4\}) = v(\{1, 2, 3, 5\}) = 26$ ,  $v(\{1, 2, 4, 5\}) = 30.002$ ,  $v(\{1, 3, 4, 5\}) = v(\{2, 3, 4, 5\}) = \frac{58}{3}$ ;  $v(N) = 38$ .  $\alpha = 14$ ,  $\beta = \frac{4}{5}$ .

Consider  $(\mathcal{P}, \mathbf{x}) = (\{\{1, 2, 3\}, \{4, 5\}\}, (9, 9, 8, 6, 6))$  and  $(\bar{\mathcal{P}}, \bar{\mathbf{x}}) = (\{\{1, 2\}, \{3, 4, 5\}\}, (9.001, 9.001, 6, \frac{20}{3}, \frac{20}{3}))$ . By calculation, we have  $\boldsymbol{\mu}(\mathcal{P}, \mathbf{x}; \alpha, \beta) = (9, 9, -6, 3.8667, 3.8667)$  and  $\boldsymbol{\mu}(\bar{\mathcal{P}}, \bar{\mathbf{x}}; \alpha, \beta) = (9.001, 9.001, -5.7341, 4.7992, 4.7992)$ . We can verify that  $(\mathcal{P}, \mathbf{x})$  belongs to the envy core by checking its stability. For instance, consider a coalition  $S = \{3, 4, 5\}$ . Suppose  $\mathbf{x}^S$  is a feasible allocation vector for  $S$ . The least utility for player 4 is  $x_4^S - \frac{\max\{0, v(\{1, 2\}) - x_4^S\}}{2} \cdot \beta$ . If  $S$  is preferred by players 3, 4, 5 over  $(\mathcal{P}, \mathbf{x})$ , then  $x_4^S - \frac{\max\{0, v(\{1, 2\}) - x_4^S\}}{2} \cdot \beta > \mu_4((\mathcal{P}, \mathbf{x}); \alpha, \beta) = 3.8667$ , from which we have  $x_4^S > 7.9$ . Similarly,  $x_5^S > 7.9$ . Since  $v(S) = \frac{58}{3}$ , we have  $x_3^S < 3.5333$ . By calculation, the utility of player 3 in  $S$  is less than  $-57.6$ . Player 3 becomes worse, and hence  $\{3, 4, 5\}$  will not form. So  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ . Similarly, we have  $(\bar{\mathcal{P}}, \bar{\mathbf{x}}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ . However,  $(\mathcal{P}, \mathbf{x}) \notin PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)))$  since  $(\bar{\mathcal{P}}, \bar{\mathbf{x}})$  dominates  $(\mathcal{P}, \mathbf{x})$  in terms of utilities. Therefore,  $(\mathcal{P}, \mathbf{x}) \notin \mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$  and  $\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) \neq \mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ .

By Theorem 4.11,  $\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) \neq \emptyset$  if and only if  $\mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) \neq \emptyset$ . It may benefit to study the nonemptiness of the envy core by reviewing the literature that investigates the nonemptiness of the core of general NTU games.

Scarf [33] finds that every balanced NTU game has a nonempty core. Billera [4] proves a more general result that every  $\pi$ -balanced NTU game has a nonempty core. Peleg and Sudhölter [29] prove that the core of an ordinal convex NTU game is nonempty. Keiding and Thorlund [21] give a necessary and sufficient balancedness condition for the nonempty core. Predtetchinski and Herings [30] extend the  $\pi$ -balancedness condition and provide a necessary and sufficient condition for the nonempty core. Compared with the condition in [21], Predtetchinski and Herings [30] avoid the construction of auxiliary games or sequences of approximating games. All the “balancedness” conditions above are from the viewpoint of utility functions. However, in a cooperative game with envy, it would be extremely complicated to express the set of possible utility vectors in each coalition. Consequently, it is hard to provide similar “balancedness” conditions showing how the payoff of each coalition affects the nonemptiness of the envy core. The conditions we provide in this section are from the perspective of payoff functions, which contributes to the cooperative game theory literature.

**5. Application.** In this section, we study a simple game and investigate how envy changes players’ behavioral decisions. Simple games are used to model voting practices in legislature and they are well studied in literature. Consider a simple game  $(N, v)$  where  $N$  is the set of players and  $v$  is the payoff function that satisfies the following [15]:

- (i):  $v(S) = 0$  or  $1$  for all  $S \subseteq N$ .
- (ii):  $v(\emptyset) = 0$  and  $v(N) = 1$  (unanimity).
- (iii):  $v(S_1) \leq v(S_2)$  for all  $S_1 \subseteq S_2 \subseteq N$  (monotonicity).
- (iv):  $v(S) + v(N \setminus S) \leq 1$  for all  $S \subseteq N$ .

We can verify that  $v$  is super-additive.

A coalition  $S$  is called a winning coalition if  $v(S) = 1$ . A player  $i$  is called a veto player if  $v(N \setminus \{i\}) = 0$ . Denote the set of all veto players as  $T$ . It is clear that  $T \subseteq S$  for all winning coalition  $S$ . Moreover, if  $v(T) = 1$ , the voting game is called an unanimity game on  $T$  [9]. The next theorem states the relation between veto players and the classic core, when envy is not considered.

**Theorem 5.1.** [29] *In a simple game  $(N, v)$ , the core is nonempty if and only if there exists a veto player. Moreover,  $\sum_{i \in T} x_i = 1$  in the core.*

Note that the grand coalition can be implicitly assumed to be formed in the core.

Envy in a simple game means a player becomes less satisfied if her/his coalition wins but s/he receives less payoff than her/his allies (in-coalition envy), or if s/he loses and thus receives less than the winning players (out-coalition envy). These concerns may change each player’s individual decision about whether to cooperate with other players and, consequently, change the outcome of the simple game.

When there is no in-coalition envy, the next theorem indicates that the non-veto players tend to stay with veto players in order to avoid the out-coalition envy, despite that all payoff is allocated to the veto players only.

**Theorem 5.2.** *In a simple game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ , if  $\alpha = 0$ , then the envy core  $C(N, v, \mu(\cdot, \cdot; 0, \beta))$  is nonempty if and only if there exists a veto player, i.e.,  $T \neq \emptyset$ . Moreover,  $\sum_{i \in T} x_i = 1$  in the envy core.*

When the in-coalition envy  $\alpha > 0$ , the non-veto players in a winning coalition may dislike that veto players take away all payoff of the coalition. In this case,

a non-veto player may prefer leaving the veto players to improve her/his utility. The next theorem shows that there exists a stable outcome of the game if the veto players can form a winning coalition.

**Theorem 5.3.** *Suppose  $\alpha > 0$  in a simple game with envy  $(N, v, \mu(\cdot, \cdot; \alpha, \beta))$ . If there exists a veto player and the set of veto players is a winning coalition, then the envy core  $\mathcal{C}(N, v, \mu(\cdot, \cdot; \alpha, \beta))$  is nonempty. Moreover,  $x_i = \frac{1}{|T|}$  for  $i \in T$  in the envy core.*

When  $T \neq \emptyset$  but  $v(T) = 0$ , the veto players need to include some of the non-veto players to form a winning coalition. In this case, a non-veto player can negotiate with the veto players for a positive payoff allocation. Otherwise, s/he may incur in-coalition envy and prefer leaving the winning coalition. The next example illustrates such a situation.

**Example 6.** In a simple game with envy,  $N = \{1, 2, 3\}$ .  $v(S) = 1$  if  $S = \{1, 2\}$ ,  $\{1, 3\}$  or  $\{1, 2, 3\}$ . Otherwise  $v(S) = 0$ .  $\alpha = 2$  and  $\beta = 2$ . In this game, the set of veto players  $T = \{1\}$  and  $v(T) = 0$ . We show that the envy core is empty.

Suppose the envy core is nonempty. By Theorem 4.1,  $(\mathcal{P}_N, \mathbf{x})$  is stable for some  $\mathbf{x}$ . Obviously,  $x_1 + x_2 + x_3 = 1$  and  $x_1, x_2, x_3 \geq 0$ . We may assume, w.l.o.g., that  $x_2 \geq x_3$ . Hence, we know  $x_3 \leq 0.5$ . We claim that  $x_1 \geq 0.5$  because, otherwise, players 1 and 3 can form a coalition  $S_1 = \{1, 3\}$  and become both better off with  $x_1^{S_1} = x_3^{S_1} = 0.5$ . For the coalition structure  $\mathcal{P}_1 = \{\{1\}, \{2, 3\}\}$ , we know  $\mu_1(\mathcal{P}_N, \mathbf{x}; 2, 2) > \underline{\mu}_1(\{1\}, 0; 2, 2)$  and  $\mu_3(\mathcal{P}_N, \mathbf{x}; 2, 2) < \underline{\mu}_3(\{2, 3\}, (0, 0); 2, 2)$ . Therefore, it must hold that  $\mu_2(\mathcal{P}_N, \mathbf{x}; 2, 2) \geq \underline{\mu}_2(\{2, 3\}, (0, 0); 2, 2)$ , which leads to  $x_1 \leq 2x_2$ . For the coalition structure  $\mathcal{P}_2 = \{\{1, 3\}, \{2\}\}$ , we know  $\mu_2(\mathcal{P}_N, \mathbf{x}; 2, 2) \geq \underline{\mu}_2(\{2\}, 0; 2, 2)$ . Therefore, for  $S_2 = \{1, 3\} \in \mathcal{P}_2$  and any feasible  $\mathbf{x}^{S_2}$ , it must hold that  $\mu_1(\mathcal{P}_N, \mathbf{x}; 2, 2) \geq \underline{\mu}_1(S_2, \mathbf{x}^{S_2}; 2, 2)$  or  $\mu_3(\mathcal{P}_N, \mathbf{x}; 2, 2) \geq \underline{\mu}_3(S_2, \mathbf{x}^{S_2}; 2, 2)$ . This leads to  $x_1 \geq 4x_2$ . Combining with  $x_1 \leq 2x_2$ , we have  $x_1 = 0$ , which becomes a contradiction. Therefore, the envy core is empty for this example.

On the other hand, if non-veto players in a winning coalition are allocated with appropriate payoffs, there can be a stable solution.

**Example 7.** In a simple game with envy,  $N = \{1, 2, 3, 4\}$ .  $v(S) = 1$  if  $S = \{1, 2\}$ ,  $\{1, 2, 3\}$ ,  $\{1, 2, 4\}$ ,  $\{1, 3, 4\}$  or  $\{1, 2, 3, 4\}$ . Otherwise  $v(S) = 0$ .  $\alpha = 1$  and  $\beta = 0$ . In this game,  $T = \{1\}$  and  $v(T) = 0$ . We show that

$$(\mathcal{P}, \mathbf{x}) = \left( \{\{1, 2\}, \{3, 4\}\}, \left( \frac{2}{3}, \frac{1}{3}, 0, 0 \right) \right)$$

is a stable solution.

By calculation, we have  $\mu_1(\mathcal{P}, \mathbf{x}; 1, 0) = \frac{2}{3}$  and  $\mu_2(\mathcal{P}, \mathbf{x}; 1, 0) = \mu_3(\mathcal{P}, \mathbf{x}; 1, 0) = \mu_4(\mathcal{P}, \mathbf{x}; 1, 0) = 0$ . Consider  $\mathcal{P}_1 = \{\{1, 2, 3\}, \{4\}\}$ . In  $\mathcal{P}_1$ , player 4's lowest utility is 0 and, therefore, s/he does not like to change  $\mathcal{P}$ . Suppose that players 1, 2 and 3 prefer  $S_1 = \{1, 2, 3\}$  over  $\mathcal{P}$ . Then the feasible payoff  $\mathbf{x}^{S_1}$  should satisfy that  $x_1^{S_1} \geq \frac{2}{3}$ ,  $x_2^{S_1} \geq \frac{2}{9}$  and  $x_3^{S_1} \geq \frac{2}{9}$ . This contradicts the fact that  $x_1^{S_1} + x_2^{S_1} + x_3^{S_1} = 1$  and, hence,  $\mathcal{P}_1$  is not better than  $(\mathcal{P}, \mathbf{x})$ . Similarly, we can prove that no other coalition structure is better than  $(\mathcal{P}, \mathbf{x})$ . Consequently,  $(\mathcal{P}, \mathbf{x})$  is stable.

Finally, when  $\alpha > 0$ , the envy core can be nonempty even if there is no veto player. The next example shows that a coalition structure may be stable when the payoff of the winning coalition is allocated properly.

**Example 8.** In a simple game with envy,  $N = \{1, 2, 3, 4, 5\}$ .  $v(S) = 1$  if  $S = \{1, 2, 3\}$ ,  $\{1, 2, 3, 4\}$ ,  $\{1, 2, 3, 5\}$ ,  $\{1, 2, 4, 5\}$ ,  $\{1, 3, 4, 5\}$ ,  $\{2, 3, 4, 5\}$  or  $\{1, 2, 3, 4, 5\}$ . Otherwise  $v(S) = 0$ .  $\alpha = 1.5$  and  $\beta = 0$ . Note that there is no veto player. We can verify that

$$(\mathcal{P}, \mathbf{x}) = \left( \{\{1, 2, 3\}, \{4, 5\}\}, \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0\right) \right)$$

is a stable solution.

**6. Conclusions.** We have proposed a new model that incorporates the envy behavior into a classic cooperative game. By quantifying a player's in-coalition envy and out-coalition envy, we transform a player's own monetary payoff to a utility function that depends on all players' payoffs. In this way, we are able to relax the traditional rationality assumption in cooperative games to accommodate behavioral preferences in players' decision making.

The new envy core characterizes stable outcomes of the game and generalizes the core of classic cooperative games. It contains coalition structures and corresponding payoff allocations that are stable under worst-case analysis of the out-coalition envy. When the in-coalition envy is no greater than the out-coalition envy, we have shown that the grand coalition is stable if the envy core is nonempty. In this case, a higher out-coalition envy may lead to larger variation of payoff allocation within the grand coalition. When the in-coalition envy is greater than out-coalition envy, players may form non-grand coalitions in a stable solution. We have also provided conditions for the envy core to be nonempty. In particular, if a game has strong coalitions and weak coalitions in terms of per-capita payoff, the envy core will be nonempty when the out-coalition envy is large enough.

As an application of this research, we investigate the envy effect in a simple game. We have found that non-veto players may gain more bargaining power against veto players due to the in-coalition envy. Consequently, the veto players have to form a winning coalition if they would like to keep their advantage over non-veto players.

Among the first in literature that quantitatively studies the effects of envy, this research provides important economic insights. Envy is one of the driving forces of the coalition formation and can help understand mechanisms behind wage structure, promotion structure, and recognition programs. Strong in-coalition envy results in social partition and, consequently, brings inefficiency to society. On the other hand, higher out-coalition envy leads the society to become a whole and to run more efficiently. In addition, fairer payoff allocations may increase the stability of a company or a team. Uneven payoff allocations could make employees feel less satisfied and join other companies. Therefore, efforts on reducing the in-coalition envy are worthwhile.

This work can be further extended in several directions. First, classic game models have the potential to be further investigated with other social preferences such as social justice and group-level comparison. For example, people may prefer joining a successful group rather than a failing group, even they receive no payoff [34]. Second, with quantitative models proposed and analyzed, human experiments may be worth designing to test and adjust these models.

**Acknowledgments.** Helpful comments from the editor and the reviewers are gratefully acknowledged. Jiahua Zhang's research was supported by Fudan University Student Growth Fund Scholarship. Shu-Cherng Fang's research was supported by

US ARO Grant # W911NF-15-1-0223. Yifan Xu's research was supported by Chinese NNSF Grant # 71531005, 71372113 and 2015BAH18F01-02. Ziteng Wang's research was supported by North Carolina State University Edward P. Fitts Fellowship and Northern Illinois University.

### Appendix: Proofs of theorems and lemmas.

**Proof of Theorem 4.1.** As  $v$  is super-additive, for any  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ , we have  $\sum_{i \in N} x_i \leq v(N)$ . Suppose  $\sum_{i \in N} x_i < v(N)$ . Then  $|S^{\mathcal{P}}(i)| < n$  for all  $i$ . Let

$$x'_i = x_i + \frac{v(N) - \sum_{i \in N} x_i}{n}, \quad i \in N.$$

Obviously,  $x'_i > x_i$ ,  $\sum_{i \in N} x'_i = v(N)$  and  $\mathbf{x}'$  is a feasible allocation of  $N$ . For all  $i \in N$  with  $1 < |S^{\mathcal{P}}(i)| < n$ , we have

$$\begin{aligned} & \underline{\mu}_i(N, \mathbf{x}'; \alpha, \beta) \\ &= x'_i - \alpha \frac{1}{n-1} \sum_{j \in S^{\mathcal{P}}(i), j \neq i} \max(x'_j - x'_i, 0) - \alpha \frac{1}{n-1} \sum_{j \notin S^{\mathcal{P}}(i)} \max(x'_j - x'_i, 0) \\ &\geq x'_i - \alpha \frac{1}{|S^{\mathcal{P}}(i)|-1} \sum_{j \in S^{\mathcal{P}}(i), j \neq i} \max(x'_j - x'_i, 0) - \beta \frac{1}{n-|S^{\mathcal{P}}(i)|} \sum_{j \notin S^{\mathcal{P}}(i)} \max(x'_j - x'_i, 0) \\ &> x_i - \alpha \frac{1}{|S^{\mathcal{P}}(i)|-1} \sum_{j \in S^{\mathcal{P}}(i), j \neq i} \max(x_j - x_i, 0) - \beta \frac{1}{n-|S^{\mathcal{P}}(i)|} \sum_{j \notin S^{\mathcal{P}}(i)} \max(x_j - x_i, 0) \\ &= \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta). \end{aligned} \tag{5}$$

Similarly, for  $i$  with  $|S^{\mathcal{P}}(i)| = 1$ ,  $\underline{\mu}_i(N, \mathbf{x}'; \alpha, \beta) > \mu_i^{\alpha, \beta}(\mathcal{P}, \mathbf{x})$ . Consequently,  $(\mathcal{P}, \mathbf{x})$  is not stable, which is a contradiction. Therefore,  $\sum_{i \in N} x_i = v(N)$ .

To prove (2), note that when  $\alpha \leq \beta$ , for any  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ , we have  $\mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta) \leq \mu_i(\mathcal{P}_N, \mathbf{x}; \alpha, \beta)$ ,  $i \in N$ . Suppose that (2) is not true. Then for some  $i_0 \in N$ ,  $\mu_{i_0}(\mathcal{P}, \mathbf{x}; \alpha, \beta) < \mu_{i_0}(\mathcal{P}_N, \mathbf{x}; \alpha, \beta) = \underline{\mu}_{i_0}(N, \mathbf{x}; \alpha, \beta)$ .

Let  $x_{i_0}'' = x_{i_0} - \epsilon$  and  $x_i'' = x_i + \frac{\epsilon}{n-1}$  where  $\epsilon > 0$ . As a piecewise linear function,  $\underline{\mu}_{i_0}(N, \mathbf{x}; \alpha, \beta)$  is continuous with respect to  $\mathbf{x}$ . Hence, there exists  $\epsilon > 0$ , s.t.,  $\underline{\mu}_{i_0}(N, \mathbf{x}''; \alpha, \beta) > \mu_{i_0}(\mathcal{P}, \mathbf{x}; \alpha, \beta)$ ,  $i \in N$ . Note that for  $i \in N \setminus \{i_0\}$ , we also have  $\underline{\mu}_i(N, \mathbf{x}''; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta)$ . Consequently,  $\underline{\mu}_i(N, \mathbf{x}''; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta)$  for all  $i \in N$ , which means  $(\mathcal{P}, \mathbf{x})$  can not be stable. Contradiction! Therefore, (2) is proved.

**Proof of Lemma 4.2.** If  $\mathcal{C}(N, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta_1)) = \emptyset$ , the lemma is obvious. If  $\mathcal{C}(N, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta_1)) \neq \emptyset$ , then for any  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta_1))$  and any coalition  $S \subseteq N$  with a feasible allocation  $\mathbf{x}^S$ , there exists  $i \in N$  such that  $\mu_i(\mathcal{P}_N, \mathbf{x}; \alpha, \beta_1) \geq \underline{\mu}_i(S, \mathbf{x}^S; \alpha, \beta_1)$ . Note that  $\beta_1 \leq \beta_2$  and  $\underline{\mu}_i(S, \mathbf{x}^S; \alpha, \beta_1) \geq \underline{\mu}_i(S, \mathbf{x}^S; \alpha, \beta_2)$ . We can verify  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta_2))$  and therefore,  $\mathcal{C}(N, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta_1)) \subseteq \mathcal{C}(N, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta_2))$ .

**Proof of Theorem 4.5.** This theorem follows Theorem 4.1, Lemma 4.2 and Theorem 4.4.

**Proof of Theorem 4.6.** Consider the following linear program (P)

$$(P) \quad \begin{aligned} \min \quad & \sum_{i \in N} x_i \\ \text{s.t.} \quad & \sum_{i \in S} x_i \geq v(S), \quad \text{for } \emptyset \neq S \subseteq N \text{ with } \frac{v(S)}{|S|} \geq v(N \setminus S) \\ & \sum_{i \in S} x_i \geq 0, \quad \text{for } \emptyset \neq S \subseteq N \text{ with } \frac{v(S)}{|S|} < v(N \setminus S) \end{aligned}$$

and its dual problem (D)

$$(D) \quad \begin{aligned} \max \quad & \sum_{\substack{S: \emptyset \neq S \subseteq N, \\ \frac{v(S)}{|S|} \geq v(N \setminus S)}} \lambda(S) v(S) \\ \text{s.t.} \quad & \sum_{\emptyset \neq S \subseteq N} \lambda(S) e^S = e^N \\ & \lambda(S) \geq 0, \quad \text{for } \emptyset \neq S \subseteq N. \end{aligned}$$

Notice that  $\lambda$  is feasible to (D) if and only if  $\lambda$  is a balanced map. It is easy to see that (P) is always feasible. Therefore, (D) is feasible and achieves the same optimal value as (P). If

$$v(N) \geq \sum_{\substack{S: \emptyset \neq S \subseteq N, \\ \frac{v(S)}{|S|} \geq v(N \setminus S)}} \lambda(S) v(S)$$

holds for all balanced map  $\lambda$ , then there exists  $\mathbf{x}$  such that  $\mathbf{x}$  is feasible to (P) and  $v(N) \geq \sum_{i \in N} x_i$ . In this case, for any  $S \subseteq N$  with  $\frac{v(S)}{|S|} < v(N \setminus S)$  and any  $\mathbf{x}^S$ , there exists  $i \in S$  such that  $x_i^S < v(N \setminus S)$ . Then  $\mu_i(\mathcal{P}_N, \mathbf{x}; 0, +\infty) \geq 0 > -\infty = \underline{\mu}_i(S, \mathbf{x}^S; 0, +\infty)$ . For any  $S$  with  $\frac{v(S)}{|S|} \geq v(N \setminus S)$  and any  $\mathbf{x}^S$ , there exists  $i \in S$  such that  $x_i \geq x_i^S$ . Then  $\mu_i(\mathcal{P}_N, \mathbf{x}; 0, +\infty) = x_i \geq x_i^S \geq \underline{\mu}_i(S, \mathbf{x}^S; 0, +\infty)$ . Therefore,  $(\mathcal{P}_N, \mathbf{x})$  is stable and the envy core is nonempty.

**Proof of Theorem 4.7.** We construct a stable solution.

First, find  $S_1 = \operatorname{argmax}_{S \subseteq N} \frac{v(S)}{|S|}$ . Second, if  $S_1 \neq N$ , let  $S_2 = \operatorname{argmax}_{S \subseteq (N \setminus S_1)} \frac{v(S)}{|S|}$ . If  $\cup_{i=1}^2 S_i \neq N$ , let  $S_3 = \operatorname{argmax}_{S \subseteq (N \setminus \cup_{i=1}^2 S_i)} \frac{v(S)}{|S|}$ . Continue this process till there exists  $1 \leq m \leq n$  such that  $\cup_{i=1}^m S_i = N$ . Let  $\mathcal{P} = \{S_1, \dots, S_m\}$  and  $x_i = \frac{v(S^{\mathcal{P}}(i))}{|S^{\mathcal{P}}(i)|}$ . We can verify that  $(\mathcal{P}, \mathbf{x})$  is a stable solution. Therefore, the envy core is nonempty.

**Proof of Theorem 4.8.** Find  $S_0 \subseteq N$  that has the largest cardinality among the coalitions  $S$  with highest in-coalition per-capita payoff  $\frac{v(S)}{|S|}$ . Let  $x_i = \frac{v(S_0)}{|S_0|}$  for  $i \in S_0$ . If  $S_0 = N$ , then  $(\mathcal{P}_N, \mathbf{x})$  is a stable solution. Otherwise, let  $x_i = \frac{v(N \setminus S_0)}{|N \setminus S_0|}$  for  $i \in N \setminus S_0$ , then  $(\{S_0, N \setminus S_0\}, \mathbf{x})$  is a stable solution.

**Proof of Theorem 4.9.** We may assume  $\beta \geq \alpha$ . Set  $x_{i_0} = v(N)$  and  $x_i = 0$  for  $i \neq i_0$ . Consider  $(\mathcal{P}_N, \mathbf{x})$ . Take any  $\emptyset \neq S \subseteq N$  and its feasible allocation  $\mathbf{x}^S$ . If  $i_0 \in S$ , then  $\underline{\mu}_{i_0}(S, \mathbf{x}^S; \alpha, \beta) \leq v(S) \leq v(N) = \mu_{i_0}(\mathcal{P}_N, \mathbf{x}; \alpha, \beta)$ . In this case, player  $i_0$  will not leave  $N$  to join  $S$ . If  $i_0 \notin S$ , then  $v(N \setminus S) \geq v(i_0) \geq \frac{v(S)}{|S|}$ . Consequently, there exists a player  $i \in S$  such that  $x_i \leq v(i_0)$ . When  $\beta$  is large enough, we have  $\underline{\mu}_i(S, \mathbf{x}^S; \alpha, \beta) \leq \frac{v(S)}{|S|} - \frac{\beta}{n-|S|}(v(i_0) - \frac{v(S)}{|S|}) \leq -\frac{\alpha}{n-1}v(N) = \mu_i(\mathcal{P}_N, \mathbf{x}; \alpha, \beta)$ . Then player  $i$  is better off when staying in  $N$ . Therefore,  $(\mathcal{P}_N, \mathbf{x})$  is a stable solution and the envy core is nonempty.

**Proof of Theorem 4.10.** Consider  $\mathcal{P}_N$  with a feasible payoff allocation  $\mathbf{x}$  such  $x_i = \frac{v(N)}{|S_0|}$  for  $i \in S_0$  and  $x_i = 0$  otherwise. Take any  $S' \subseteq N$  and its feasible  $\mathbf{x}^{S'}$ . If  $S' \cap S_0 \neq \emptyset$ , then there exists  $i \in S' \cap S_0$  such that  $\underline{\mu}_i(S', \mathbf{x}^{S'}; \alpha, \beta) \leq \frac{v(S')}{|S' \cap S_0|} \leq \frac{v((S' \cap S_0) \cup (N \setminus S_0))}{|S' \cap S_0|} < \frac{v(S_0)}{|S_0|} \leq \frac{v(N)}{|S_0|} = \mu_i(\mathcal{P}_N, \mathbf{x}; \alpha, \beta)$ . Hence, player  $i$  would rather stay in  $N$  instead of  $S'$ . If  $S' \cap S_0 = \emptyset$ , then for sufficiently large  $\beta$ , noting that  $v(N \setminus S') \geq v(S_0) > \frac{|S_0|}{|S|} v(S \cup (N \setminus S_0)) \geq v(N \setminus S_0) \geq v(S')$  for all  $S \subsetneq S_0$ , there exists  $i \in S'$  such that  $\underline{\mu}_i(S', \mathbf{x}^{S'}; \alpha, \beta) \leq \frac{v(S')}{|S'|} - \frac{\beta}{n-|S'|} (v(N \setminus S') - \frac{v(S')}{|S'|}) \leq -\frac{\alpha}{n-1} v(N) = \mu_i(\mathcal{P}_N, \mathbf{x}; \alpha, \beta)$ . Therefore,  $(\mathcal{P}_N, \mathbf{x})$  is a stable solution and the envy core is nonempty.

**Proof of Theorem 4.11.** For any  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ , pick  $S = N$ . It holds that  $\nexists (\bar{\mathcal{P}}, \bar{\mathbf{x}}) \in \Omega$  such that  $\mu_i(\bar{\mathcal{P}}, \bar{\mathbf{x}}; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta)$  for  $i \in N$ . It is easy to see that  $\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) \subseteq \Omega$ . Therefore,  $(\mathcal{P}, \mathbf{x}) \in PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)))$ , which means  $\mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)) \subseteq PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)))$ .

For any  $(\mathcal{P}, \mathbf{x}) \in PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)))$ , suppose  $(\mathcal{P}, \mathbf{x}) \notin \mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ . Then  $\exists S \subseteq N$  and  $\exists (\bar{\mathcal{P}}, \bar{\mathbf{x}}) \in V(S)$  such that  $\mu_i(\bar{\mathcal{P}}, \bar{\mathbf{x}}; \alpha, \beta) > \mu_i(\mathcal{P}, \mathbf{x}; \alpha, \beta)$  for  $i \in S$ . Note that  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$  and then we have  $S = N$ . We can verify that  $(\bar{\mathcal{P}}, \bar{\mathbf{x}}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ . Therefore,  $(\bar{\mathcal{P}}, \bar{\mathbf{x}})$  dominates  $(\mathcal{P}, \mathbf{x})$ , which contradicts with  $(\mathcal{P}, \mathbf{x}) \in PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta)))$ . Thus  $(\mathcal{P}, \mathbf{x}) \in \mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ , which means  $PF(\mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))) \subseteq \mathcal{C}(N, \Omega, V, \boldsymbol{\mu}(\cdot, \cdot; \alpha, \beta))$ .

**Proof of Theorem 5.2.** The sufficiency follows Theorem 5.1 and Lemma 4.2. For the necessity, consider a stable solution  $(\mathcal{P}_N, \mathbf{x}) \in \mathcal{C}(N, v, \boldsymbol{\mu}(\cdot, \cdot; 0, \beta))$ . There exists a player  $i_0$  such that  $x_{i_0} > 0$ . Let  $S = N \setminus \{i_0\}$ . If  $i_0$  is not a veto player, we have  $v(S) = 1$  and  $v(\{i_0\}) = 0$ . Let  $x_i^S = x_i + \frac{x_{i_0}}{n-1}$  for  $i \in S$ . Then  $\mathbf{x}^S$  is a feasible payoff allocation of  $S$  and  $\underline{\mu}_i(S, \mathbf{x}^S; 0, \beta) = x_i^S > x_i = \mu_i(\mathcal{P}_N, \mathbf{x}; 0, \beta)$  for  $i \in S$ . This contradicts the stability of  $(\mathcal{P}_N, \mathbf{x})$ . Hence,  $i_0$  is a veto player. Actually, only veto players get positive payoff allocation. Therefore,  $\sum_{i \in T} x_i = 1$ .

**Proof of Theorem 5.3.** Let  $x_i = \frac{1}{|T|}$  for  $i \in T$  and  $x_i = 0$  for  $i \in N \setminus T$ . If  $0 < \alpha \leq \beta$ , then  $(\mathcal{P}_N, \mathbf{x})$  is stable. If  $0 \leq \beta \leq \alpha$ , then  $\left(\{T, \{i_1\}, \dots, \{i_{|N \setminus T|}\}\}, \mathbf{x}\right)$ ,  $i_k \in N \setminus T$ ,  $k = 1, \dots, |N \setminus T|$ , is a stable solution. Therefore, the envy core is nonempty.

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Received January 2016; 1st revision June 2016; final revision September 2016.

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