

RESEARCH ARTICLE

Inventory transshipment game with limited supply: Trap or treat

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Abstract

Inventory transshipment is generally shown to be beneficial to retailers by matching their excess demand with surplus inventory. We investigate an inventory transshipment game with two newsvendor-type retailers under limited total supply and check whether the retailers are better off than the case without transshipment. We derive the ordering strategies for the retailers and show that unlike the unlimited supply case, a pure Nash equilibrium only exists under certain conditions. Furthermore, contrary to the conventional wisdom, we show that inventory transshipment may not always benefit both retailers. Although one of the retailers is guaranteed to be better off, the other could be worse off. The decision criteria are then provided for the retailers to determine if they will benefit from the exercise of inventory transshipment. Numerical study indicates that the carefully chosen transshipment prices play an important role in keeping inventory transshipment beneficial to both retailers. Subsequently, a coordinating mechanism is designed for the retailers to negotiate transshipment prices that maximize the total profit of the two retailers while keeping each of them in a beneficial position.

KEYWORDS

bargaining, game theory, inventory transshipment, newsvendor, supply chain coordination

1 | INTRODUCTION

Inventory transshipment refers to the practice that the surplus inventory of a retailer is transshipped to another retailer with excess demand. As a pooling strategy, inventory transshipment helps retailers to mitigate the risk of demand uncertainty and provides them a second chance to satisfy their demand. Inventory transshipment is commonly conducted in industries including apparel and fashion goods (Bitran et al., 1998), automobile dealership (Zhao and Atkins, 2009), transportation and maritime, service parts (Zhao et al., 2005), soft drink (Dong et al., 2012), toys and electronics by companies such as *Foot Locker*, *Ingram Micro*, *Teknosa* (Özdemir et al., 2013), *Caterpillar*, *General Motors* (Zhao et al., 2005), *Grainger*

Integrated Supply Operations (Narus and Anderson, 1996) and many others. Inventory transshipment has received extensive study in the past decade. Paterson et al. (2011) and Huang (2013) provide excellent reviews on general inventory transshipment models such as proactive (before demand observation) and reactive (after demand observation) transshipment, lateral and vertical transshipment, transshipment in centralized and decentralized supply chains, and so on. In this paper, we consider the reactive lateral inventory transshipment between decentralized retailers in the same echelon of a supply chain.

Inventory transshipment can be helpful not only when the retailers face demand uncertainty, but also when the supply capacity is limited and the retailers may not be able to

stock the desired levels of inventory. For example, in industries like apparel, toys, automobiles and personal computers, the supplier's capacity may not be sufficient to fulfill all orders of the retailers because it could be expensive or infeasible to increase the supply in response to the substantial demand change in a short term (Cachon and Lariviere, 1999b; Deshpande and Schwarz, 2002). Consequently, a supplier has to allocate her limited supply to the retailers according to an inventory allocation rule. Under such circumstances, chances could be higher that a retailer will have insufficient inventory and unmet demand. Inventory transshipment would then become highly necessary.

In reality, many retailers face the limited supply on the supply side and conduct inventory transshipment on the demand side. For example, in the European System of Central Banks, the member banks (retailers) not only borrow cash from the European Central Bank (supplier) whose money supply is tight and only available in a weekly or monthly basis, but also borrow from other banks (transshipment) to meet their short-term needs (Özdemir et al., 2013). Another example is the capacity management of scarce resource in disaster and pandemic response. In response to the demand surge for scarce healthcare resources such as beds, vaccines, medicines, nursing staff and doctors in an influenza pandemic, a recommended strategy is regional "mutual aid" (transshipment) among counties, states and the nationwide Strategic National Stockpiles that Center for Disease Control and Prevention maintains (see Arora et al., 2010, and references therein). In those situations, the decision problems of the retailers are complicated by the competition on the supply side and the cooperation on the demand side.

We find that existing literature on inventory transshipment often implicitly assumes that the retailers' inventory stocks are not constrained by the supply availability, which is not necessarily true in business practice. As a result, the profit maximization decision of a retailer conducting inventory transshipment and facing limited supply is not fully analyzed and the impact of inventory transshipment on a capacitated supply chain is not fully unveiled. Some remaining questions include: Given the limited supply and a general allocation rule, what are the retailers' equilibrium order decisions? More importantly, is inventory transshipment beneficial to all retailers? If not, how can a retailer determine whether to participate in an inventory transshipment contract would benefit himself? Can the retailers be coordinated?

In this paper, we attempt to answer the above research questions by studying two decentralized retailers who place orders to a capacitated supplier before observing their stochastic demand and conduct inventory transshipment after demand uncertainty is resolved. Unlike the unlimited supply case in which the existence and uniqueness of the Nash equilibrium is guaranteed, we find that when the supply is limited, a pure Nash equilibrium only exists under certain conditions, and may not be unique, due to retailers' competition for the supply and cooperation through inventory transshipment.

Furthermore, with the existence of the equilibrium, we investigate the benefit of inventory transshipment under the limited supply by comparing each retailer's equilibrium profit with a baseline—the equilibrium profit of the game with the same supply but without inventory transshipment. An important finding is that only one of the retailers, but not always both, is guaranteed to be better off. In other words, inventory transshipment do not necessarily benefit both retailers when the supply capacity is limited. A retailer can check certain conditions, provided in this paper, to know whether he will be better off and accordingly, decide whether to participate in inventory transshipment before placing orders. Following these conditions, we show that inventory transshipment can benefit both retailers only in certain scenarios, for example, when each retailer receives his preferred amount or a larger allocation of the supply than that in the baseline game. However, if a retailer receives lower equilibrium inventory than that in the no-transshipment baseline game due to supply competition, he will unlikely meet his demand and consequently, lose profit that could have been generated by more initial inventory. In this scenario, if this profit loss is not fully recovered through inventory transshipment, this retailer will become worse off. We find that the retailers' demands and cost structures play important roles in this new finding. In particular, our numerical study indicates that the transshipment prices affect the allocation of transshipment profit and thus should be carefully determined to make sure that inventory transshipment always benefits both retailers. To achieve this goal, we provide a coordinating mechanism, by allowing the transshipment prices to be negotiated by the retailers, to ensure that both retailers are better off and the total retailer profit is maximized at the Nash equilibrium.

The main contributions of this paper are as follows.

1. This paper provides retailers with profit-maximizing order and inventory transshipment strategies in a realistic yet less studied scenario of limited supply capacity. Specifically, we explicitly characterize the Nash equilibrium orders of two retailers. Moreover, we suggest that a retailer should check certain conditions derived in this paper to determine whether participating in inventory transshipment indeed benefits himself. Analysis and obtained results throughout this paper are applicable to asymmetric retailers facing general stochastic demands and a broad class of capacity allocation rules.
2. This research reveals new managerial insights into the benefit of inventory transshipment in capacitated decentralized supply chains. When supply is limited, one of the retailers will be better off but both may not. The predetermined transshipment prices are identified as an important cause. A coordinating mechanism is designed to ensure that both retailers benefit from inventory transshipment and to

maximize their total profit if a first best supply chain outcome is desired.

3. This research provides a modeling platform for investigating a broad spectrum of supply chain decisions involving inventory transshipment and limited supply capacity. Among the first to consider such problems, this paper develops extendable analytic tools for further analyzing how inventory transshipment and limited supply affect retailers' decisions and the supply chain performance in more general environments. The two-newsvendor model and its insights in this paper may serve as baselines in comparison with other related problems.

The rest of this paper is organized as follows. Section 2 reviews relevant literature. Section 3 introduces the proposed game model of two newsvendors with inventory transshipment under limited supply. Section 4 provides game-theoretic analysis and an illustrative example. Section 5 investigates how the limited supply capacity impacts the benefit of inventory transshipment. Section 6 proposes a coordinating mechanism. Section 7 concludes the paper. All proofs are in the Appendix A. Extended discussion can be found in Appendices B to F.

2 | RELATED LITERATURE

This paper is closely related to two streams of literature: inventory transshipment and capacity allocation. The vast majority of literature on inventory transshipment assumes unlimited capacity of the supplier which allows the retailers to stock freely the desired amounts of inventory. Those that investigate the inventory decision and profit-related benefit of retailers in a decentralized supply chain are most relevant and thus reviewed in this section. Readers are referred to Paterson et al. (2011) and Huang (2013) for more comprehensive reviews on inventory transshipment. On the other hand, research on capacity allocation tends to consider retailers that are monopolies or are involved in demand competition, rather than inventory transshipment.

Rudi et al. (2001) consider the inventory decisions of two retailers who conduct inventory transshipment after their stochastic demand is realized. They show that a Nash equilibrium exists and is unique for two decentralized retailers, and by comparing with the centralized first-best inventory levels, they investigate the possibility of using the predetermined transshipment prices to coordinate the retailers. Hu et al. (2007) provide sufficient and necessary conditions for such coordinating transshipment prices to exist. They allow each retailer to have uncertain yet independent capacities, and in subsequent studies, Hu et al. (2008) establish the retailers' joint inventory and transshipment policies in a centralized system and Chen et al. (2015) simplify the analysis with $L^\#$ -convexity. Rong et al. (2010) consider two decentralized

retailers who transship inventory between two demand sub-periods and show that a coordinating transshipment price does not exist. Relaxing the setting of fixed transshipment prices, Hezarkhani and Kubiak (2010) design a coordinating contract which induces the decentralized retailers to negotiate a transshipment pricing mechanism that lead to maximizing their total profit at the Nash equilibrium of inventory levels. Hanany et al. (2010) design a transshipment fund mechanism for coordinating the system by introducing a third-party that facilitates the redistribution of transshipment profits. Zhao and Atkins (2009) find that the transshipment price is an important factor (along with retailer differentiation) for a retailer to decide whether he prefers inventory transshipment or demand substitution. Including the manufacturer in the system, Dong and Rudi (2004) show that the manufacturer can benefit when dealing with centralized retailers who conduct inventory transshipment. Moreover, Shao et al. (2011) show that the manufacturer prefers a higher transshipment price and dealing with centralized retailers, whereas the decentralized retailers prefer a lower transshipment price. Lee and Park (2016) consider two identical retailers who are supplied by a single supplier with uncertain capacity and show that the existence of coordinating transshipment prices is further limited due to the uncertain supply capacity.

Besides the transshipment price, other allocation methods to distribute the profit generated by inventory transshipment are studied. Anupindi et al. (2001) consider a general transshipment system of multiple retailers and use a centralized transportation problem to determine the transshipment pattern. They propose to use the optimal dual solutions of this transportation problem to allocate the profit of transshipment and prove that the retailer profits allocated by this rule is in the core of the cooperative transshipment game. Granot and Sošić (2003) find that under the dual allocation rule, if retailers have the option of deciding the portion of surplus inventory and excess demand to share, they may tend to hold back or even not to share their surplus inventory at all. Nevertheless, Huang and Sošić (2010a) find that if the inventory transshipment game is repeated in infinitely many periods, the retailers should share all residual inventory and demands. Moreover, Huang and Sošić (2010b) compare the dual allocation and transshipment price and find that neither allocation method dominates the other.

The most relevant literature on capacity allocation includes those on the retailers' strategic ordering behaviors and the supply chain performance under different allocation rules. Lee et al. (1997) show that the retailers tend to inflate their order quantities knowing that the supplier's capacity is uncertain, which contributes to the bullwhip effect. Assuming each retailer is a local monopoly with linear price-dependent demand, Cachon and Lariviere (1999a) show that proportional and linear allocation rules are manipulatable that lead the retailers to inflate their orders. Uniform allocation, on the other hand, is a "truth-telling" rule that eliminates the order inflation. Nevertheless, Cachon and Lariviere (1999b) show

that the truth-telling rules can lower profits of the supplier, the retailers and the supply chain. The supplier can choose a higher level of capacity under a manipulable allocation rule than under a truth-telling rule. When the retailers are involved in demand competition, Liu (2012) shows that uniform allocation becomes manipulable and subsequently, Cho and Tang (2014) propose a truth-telling “competitive allocation” rule. We refer the readers to Lariviere (2010) for a concise review of capacity allocation.

This paper considers two decentralized retailers who are faced with the limited capacity of their common supplier and conduct inventory transshipment after the demand uncertainty is resolved. With optimal ordering strategies and the Nash equilibrium derived for the retailers, we investigate one of the fundamental questions of inventory transshipment: Will the retailers be better off, at the Nash equilibrium of ordering decisions with transshipment, than what they would be with no transshipment? To the best of our knowledge, this inventory transshipment game with limited supply has not been studied in the literature. Furthermore, we propose a transshipment price-based coordinating mechanism to maximize the retailers’ total profit.

3 | MODELS

We consider two newsvendor-type retailers who purchase a product from the same supplier before their stochastic demands are realized. The supplier has limited capacity $K > 0$. If the total order quantity of the newsvendors exceeds K , the supplier will allocate her capacity to the newsvendors by an inventory allocation rule. Let the newsvendors be indexed by i and j with $i, j = 1, 2$ and $i \neq j$. Denote newsvendor i ’s order quantity by $y_i \geq 0$ and the inventory allocation he receives by $x_i \geq 0$. Let $\mathbf{y} = (y_1, y_2)$ and $\mathbf{x} = (x_1, x_2)$. The inventory allocation rule is defined as a continuous function $\boldsymbol{\varphi}$ that assigns an inventory allocation vector $\mathbf{x}$ to each order vector $\mathbf{y}$. Denote $\mathbf{x} = \boldsymbol{\varphi}(\mathbf{y}) = (\varphi_1(y_1, y_2), \varphi_2(y_2, y_1))$. We assume that $\boldsymbol{\varphi}$ satisfies the following conditions:

- (I) (Feasible condition) $\varphi_1(y_1, y_2) + \varphi_2(y_2, y_1) \leq K$ and $0 \leq \varphi_i(y_i, y_j) \leq y_i$ for $i, j = 1, 2, i \neq j$.
- (II) (Efficient condition) If $y_1 + y_2 \leq K$, then $\varphi_i(y_i, y_j) = y_i$ for $i, j = 1, 2, i \neq j$. If $y_1 + y_2 > K$, then $\varphi_1(y_1, y_2) + \varphi_2(y_2, y_1) = K$.
- (III) (Increasing condition) $\varphi_i(\tilde{y}_i, y_j) \geq \varphi_i(y_i, y_j)$ for all $\tilde{y}_i > y_i$ and $y_j, i, j = 1, 2, i \neq j$.
- (IV) (Individually responsive condition) If $0 < \varphi_i(y_i, y_j) < K$ and $\tilde{y}_i > y_i$, then $\varphi_i(\tilde{y}_i, y_j) > \varphi_i(y_i, y_j)$ for $i, j = 1, 2, i \neq j$.
- (V) (Fair condition) If $y_i \geq y_j$, then $\varphi_i(y_i, y_j) \geq \varphi_j(y_j, y_i)$ for $i, j = 1, 2, i \neq j$.

Conditions (I)–(IV) are adopted from Cachon and Lariviere (1999b). Condition (I) requires that the total inventory allocated is no more than the available supply capacity and

a newsvendor cannot receive more than his order quantity. Condition (II) ensures that no supply capacity is wasted. Condition (III) guarantees that a newsvendor never receives less by ordering more. Condition (IV) means that if a newsvendor receives a positive allocation and orders more, he will definitely get more unless he has already been allocated all of the capacity. Condition (V) means that whoever places a larger order receives a larger portion of the supply. Commonly used rules that satisfy the conditions (I)–(V) include the proportional rule

$$\varphi_i(y_i, y_j) = \begin{cases} y_i, & \text{if } y_i + y_j \leq K, \\ \frac{y_i}{y_i + y_j} K, & \text{if } y_i + y_j > K. \end{cases}$$

and the linear rule when $y_i \leq K, i = 1, 2$,

$$\varphi_i(y_i, y_j) = \begin{cases} y_i, & \text{if } y_i + y_j \leq K, \\ y_i - \frac{y_i + y_j - K}{2}, & \text{if } y_i + y_j > K. \end{cases}$$

We point out that an allocation rule satisfying conditions (I)–(V), as we will see in Section 4, is not “truth-telling” in the sense that it induces the gaming effect between newsvendors: they tend to inflate their orders so as to receive larger allocations. This is largely due to the individually responsive condition (IV). Nevertheless, we keep this condition for two reasons. First, it is unclear if there exists a truth-telling allocation rule that can eliminate the gaming effect due to supply competition in our model. Known truth-telling rules such as uniform allocation rule (Cachon and Lariviere, 1999a,b) and competitive allocation rule (Cho and Tang, 2014) are only proved to eliminate the gaming effect of two retailers in a Cournot competition. The newsvendors in this paper face exogenous stochastic demands and extra complication in ordering decisions due to inventory transshipment. Second, the existence and characteristics of a truth-telling rule are not among the research questions in this paper. Actually, we intend to investigate the gaming effect of capacity allocation and its interplay with inventory transshipment on a newsvendor’s decision-making.

The sequence of events is illustrated in Figure 1. The supplier announces the supply capacity K and the inventory allocation rule $\boldsymbol{\varphi}$ before the newsvendors place orders. Hence, K and $\boldsymbol{\varphi}$ are exogenous for the newsvendors. The newsvendors are not allowed to place arbitrarily large orders. Without loss of generality, we assume that the order limit is K for each newsvendor. Setting unequal order limits does not invalidate the results of this study (see Appendix B).

The newsvendors pay unit wholesale prices w_i for the received inventories and sell to customers at retail prices $r_i, i = 1, 2$. We assume that the customer demand is stochastic and continuously distributed. Let D_i denote the demand of the newsvendor i with a cumulative distribution function F_{D_i} known to both newsvendors. Assume F_{D_i} is strictly increasing in the support of D_i . Similar to Rudi et al. (2001), D_1 and D_2 are not required to be mutually independent. After the demands at both newsvendors are realized, inventory

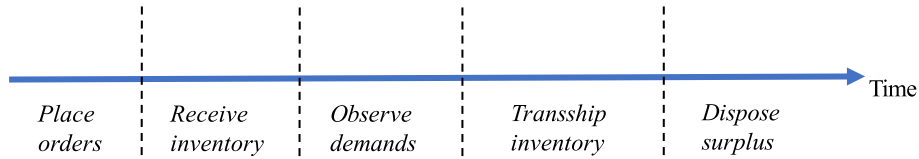


FIGURE 1 Sequence of events in Model NTL [Colour figure can be viewed at wileyonlinelibrary.com]

transshipment takes place. That is, if the newsvendor i has unsatisfied demand and the newsvendor j has surplus inventory, then the newsvendor i will purchase inventory from the newsvendor j at a transshipment price c_{ji} and sell at r_i . (The subscript ji indicates that the surplus inventory is transshipped from j to i .) Without loss of generality, we assume that the newsvendor j incurs a unit cost of t_{ji} for handling and shipping the surplus inventory transshipped from j to i . In a single-period setting, leftover inventory at the newsvendors will be disposed at salvage values s_i and s_j , respectively. In this model, w_i , r_i , s_i , c_{ij} and t_{ij} , $i, j = 1, 2$, $i \neq j$, are exogenous for both newsvendors and their relations are assumed as follows. Extension to endogenous wholesale prices and supplier's decision is in Appendix D.

Assumption 1 Assume that the price parameters w_i , r_i , s_i , c_{ij} and t_{ij} , $i, j = 1, 2$, $i \neq j$ satisfy the following conditions:

- (1) $r_i > w_i > s_i$, $i = 1, 2$;
- (2) $r_i \geq c_{ji} \geq s_j + t_{ji}$, $i, j = 1, 2$, $i \neq j$;
- (3) $r_i \geq c_{ij}$, $i, j = 1, 2$, $i \neq j$;
- (4) $w_i < w_j + t_{ji}$, $s_i < s_j + t_{ji}$ and $r_i < r_j + t_{ji}$, $i, j = 1, 2$, $i \neq j$.

Condition (1) is the basic assumption of the newsvendor model. Condition (2) ensures that each newsvendor receives a nonnegative portion of the profit generated by inventory transshipment. Condition (3) implies that a newsvendor prioritizes his customer over the other newsvendor. Condition (4) indicates that inventory transshipment occurs only when one newsvendor has excess demand and the other has surplus inventory (Tagaras, 1989).

To clearly unveil the effect of inventory transshipment, we assume that the transshipment prices c_{ij} and c_{ji} are predetermined and exogenous. This setting is adopted from Rudi et al. (2001) and motivated by the fact that it is seen in practice including car dealership, maritime industry and railroad and motor industry (see Zhao and Atkins, 2009; Shao et al., 2011, and references therein). In this way, each unit of transshipped inventory from j to i generates an extra profit of $r_i - t_{ji} - s_j$ out of which the newsvendor i receives $r_i - c_{ji}$ and the newsvendor j receives $c_{ji} - t_{ji} - s_j$. Such allocation of transshipment profit is convenient for implementation and in certain scenarios (Huang and Sošić, 2010b) works better than other profit allocation methods, for example, dual allocation. Moreover, a pair of carefully set transshipment prices may also coordinate the newsvendors (Hu et al., 2007). We keep this setting throughout this paper except in Section 6 where the newsvendors are

allowed to determine the transshipment prices in a suggested coordinating mechanism.

Both newsvendors are assumed to have perfect information, and they are rational with respective objectives to maximize their own profits by deciding order quantities unilaterally and simultaneously. We call this noncooperative game *Newsvendor game with inventory Transshipment under Limited supply* (Model NTL). We also consider two variants of the game. The first is *Newsvendor game under Limited supply* (Model NL) in which inventory transshipment is not allowed and the supply capacity remains limited. We will use this as a baseline case to study the benefit of inventory transshipment in Section 5. The second is *Newsvendor game with inventory Transshipment* (Model NT) in which the supply capacity is unlimited. This model has been well studied in literature. We will use some existing results of Model NT in the analysis of Model NTL in Section 4. Extension to multiple newsvendors is discussed in Appendix F.

4 | ANALYSIS

The supply capacity constraint and the inventory transshipment may complicate a newsvendor's ordering strategy due to their opposite effects. Generally, inventory transshipment may lower a newsvendor's order quantity by pooling his risk with the other newsvendor. On the other hand, competition for the limited supply may induce a newsvendor to increase his order. In this section, we investigate newsvendors' optimal decisions and the Nash equilibrium in Model NTL. We begin with Model NL and Model NT because they demonstrate the individual effects of limited supply and inventory transshipment separately.

4.1 | Order decisions and equilibrium in model NL

Consider newsvendor i . Let $\pi_i(x_i)$ denote the classic newsvendor profit function with respect to his inventory level x_i . That is, $\pi_i(x_i) = -w_i x_i + r_i \mathbf{E}[\min\{D_i, x_i\}] + s_i \mathbf{E}[(x_i - D_i)^+]$ where $\mathbf{E}[\cdot]$ is the expectation of a random variable and $(\cdot)^+ = \max\{\cdot, 0\}$. We know that $\pi_i(x_i)$ is concave in x_i and reaches its maximum at x_i^* such that $F_{D_i}(x_i^*) = \frac{r_i - w_i}{r_i - s_i}$ if supply is unlimited. In the presence of supply capacity constraint, however, newsvendor i 's profit depends not only on y_i but y_j . We use $\Pi_i^L(y_i, y_j) = \pi_i(\varphi_i(y_i, y_j))$ to denote the expected profit of newsvendor i when he orders y_i and newsvendor j orders y_j . By the increasing condition (III) of φ and the concavity

TABLE 1 Equilibrium orders and inventory allocation in Model NL

Case	$\mathbf{y}^{N,L}$	$\mathbf{x}^{N,L}$
$K > x_1^* + x_2^*$	(x_1^*, x_2^*)	(x_1^*, x_2^*)
$K = x_1^* + x_2^*$	$(y_1, \psi_2(y_1 x_2^*)), x_1^* \leq y_1 \leq K$	(x_1^*, x_2^*)
$K < x_1^* + x_2^*$ $x_1^* < \frac{K}{2}$	$(\psi_1(K x_1^*), K)$	$(x_1^*, K - x_1^*)$
$x_2^* < \frac{K}{2}$	$(K, \psi_2(K x_2^*))$	$(K - x_2^*, x_2^*)$
$\min\{x_1^*, x_2^*\} \geq \frac{K}{2}$	(K, K)	$(\frac{K}{2}, \frac{K}{2})$

of $\pi_i(x_i)$, $\Pi_i^L(y_i, y_j)$ is quasi-concave with respect to y_i for any given y_j .

The best response function $R_i^L(y_j)$ is defined as newsvendor i 's optimal order quantity that maximizes $\Pi_i^L(y_i, y_j)$ if newsvendor j orders y_j . If $x_i^* + y_j \leq K$, newsvendor i simply orders x_i^* . If $x_i^* + y_j > K$, he tends to increase order quantity to receive x_i^* or as close to x_i^* as possible. More specifically, if $x_i^* \leq \varphi_i(K, y_j)$ which is the maximum amount of inventory that newsvendor i can receive, then $R_i^L(y_j)$ should satisfy $\varphi_i(R_i^L(y_j), y_j) = x_i^*$. To better characterize this, for given x_i and y_j such that $x_i \leq \varphi_i(K, y_j)$, let $\psi_i(y_j | x_i)$ denote the solution to $\varphi_i(y_i, y_j) = x_i$ with respect to y_i . Such $\psi_i(y_j | x_i)$ exists thanks to conditions (III) and (IV). Lastly, if $x_i^* > \varphi_i(K, y_j)$, then newsvendor i orders K which is the largest order allowed. Summarizing the analysis above, we have

$$R_i^L(y_j) = \begin{cases} x_i^*, & \text{if } y_j + x_i^* \leq K, \\ \psi_i(y_j | x_i^*), & \text{if } K - y_j < x_i^* \leq \varphi_i(K, y_j), \\ K, & \text{if } x_i^* > \varphi_i(K, y_j). \end{cases}$$

A pair of orders (y_1, y_2) is a Nash equilibrium of Model NL if $y_1 = R_1^L(y_2)$ and $y_2 = R_2^L(y_1)$. There exists a Nash equilibrium of orders in Model NL because for each newsvendor, the decision space $[0, K]$ is compact and the profit function is quasi-concave and continuous (see Fudenberg and Tirole, 1991, Theorem 1.2). The equilibrium orders $\mathbf{y}^{N,L}$ and the resulted inventory allocation $\mathbf{x}^{N,L} = \boldsymbol{\varphi}(\mathbf{y}^{N,L})$ can be calculated by checking $R_1^L(y_2)$ and $R_2^L(y_1)$. The calculation is straightforward and we list the results in Table 1. We see that a newsvendor tends to inflate his order quantity, by gaming the allocation rule, with the goal of receiving his optimal inventory level x_i^* . The newsvendor with the smaller optimal inventory level ($x_i^* < \frac{K}{2}$) is favored because his inflated order quantity is less likely to be restricted by the order limit K . The Nash equilibrium of orders $\mathbf{y}^{N,L}$ is unique except when $x_1^* + x_2^* = K$. Nevertheless, the inventory allocation $\mathbf{x}^{N,L}$ at the Nash equilibrium is always unique regardless of K .

4.2 | Existing results of model NT

In Model NT, the supply is unlimited and thus, $\mathbf{x} = \mathbf{y}$. Let $\pi_i^T(x_i, x_j)$ denote the expected profit of newsvendor i with respect to inventory decisions x_i and x_j . That is,

$$\pi_i^T(x_i, x_j) = -w_i x_i + r_i \mathbf{E}[\min\{D_i, x_i\}] + s_i \mathbf{E}[(x_i - D_i)^+]$$

$$+ (c_{ij} - t_{ij} - s_i) \mathbf{E}[\min\{(x_i - D_i)^+, (D_j - x_j)^+\}] \\ + (r_i - c_{ji}) \mathbf{E}[\min\{(D_i - x_i)^+, (x_j - D_j)^+\}]$$

where the fourth term is the extra profit of transshipping inventory to the newsvendor j , and the fifth term is the extra profit of receiving inventory from the newsvendor j and selling to customers.

Rudi et al. (2001) prove that $\pi_i^T(x_i, x_j)$ is concave in x_i and there exists a unique Nash equilibrium of inventory decisions in Model NT. The newsvendor i 's best response $R_i^T(x_j)$ decreases in x_j , which is opposite to the increasing $R_i^L(y_j)$ in Model NL. This indicates that inventory transshipment and the limited supply have different effects on a newsvendor's order decision. Coordination between the newsvendors in Model NT, which means that the equilibrium $\mathbf{x}^{N,T}$ maximizes their total profit, can be achieved if transshipment prices are properly set under certain conditions (Hu et al., 2007).

4.3 | Order decisions and equilibrium in Model NTL

In Model NTL, a newsvendor's ordering strategy becomes more complex than a hybrid of that in Model NL and Model NT. Let $\Pi_i^{TL}(y_i, y_j)$ denote the expected profit of newsvendor i with respect to order quantities y_i and y_j . That is,

$$\Pi_i^{TL}(y_i, y_j) = \pi_i^T(\varphi_i(y_i, y_j), \varphi_j(y_j, y_i)) \\ = \begin{cases} \pi_i^T(y_i, y_j), & \text{if } y_i + y_j \leq K, \\ \pi_i^T(\varphi_i(y_i, y_j), K - \varphi_i(y_i, y_j)), & \text{if } y_i + y_j > K. \end{cases}$$

The newsvendor i seeks his optimal y_i that maximizes $\Pi_i^{TL}(y_i, y_j)$. Unlike Model NL and Model NT, $\Pi_i^{TL}(y_i, y_j)$ is not necessarily quasi-concave for any given y_j . This is because when the supply is fully allocated, the profit $\pi_i^T(x_i, K - x_i)$ may not necessarily be quasi-concave in x_i under the most general settings of price parameters and demand distributions. Nevertheless, we observe that $\pi_i^T(x_i, K - x_i)$ is quasi-concave in most experimental instances that we tested (see Appendix C). In general, there is no easy condition that separates the price parameters and demand distributions for the quasi-concavity check of $\pi_i^T(x_i, K - x_i)$. Nevertheless, we provide a special case below in which $\pi_i^T(x_i, K - x_i)$ is not only quasi-concave, but actually concave.

Proposition 1 *If both D_i and D_j are uniformly distributed in $[0, u]$ with $u \geq K$, and the price*

parameters satisfy $c_{ij} - t_{ij} - s_i \leq \frac{r_i - s_i}{2} \leq c_{ji} - s_i$, then $\pi_i^T(x_i, K - x_i)$ is concave in $[0, K]$.

In this special case, we consider uniform distributions with an upper bound no less than K as a possible scenario where the supply capacity is limited compared with the demand. To interpret the condition $c_{ij} - t_{ij} - s_i \leq \frac{r_i - s_i}{2} \leq c_{ji} - s_i$, firstly we note that $c_{ij} - t_{ij} - s_i$ is the share of transshipment profit of each unit of surplus inventory from newsvendor i to newsvendor j . Secondly, $c_{ji} - s_i$ is newsvendor i 's net risk of each unit of inventory transshipped from newsvendor j to newsvendor i because if newsvendor i buys a unit from newsvendor j but fails to sell, then newsvendor i pays the transshipment price c_{ji} and only receives a salvage value s_i . Thirdly, $r_i - s_i$ represents the net market value of the product (retail price r_i with the salvage value s_i subtracted). Therefore, if we view $\frac{r_i - s_i}{2}$ as a threshold, the meaning of the condition $c_{ij} - t_{ij} - s_i \leq \frac{r_i - s_i}{2} \leq c_{ji} - s_i$ is twofold: (a) $c_{ij} - t_{ij} - s_i \leq \frac{r_i - s_i}{2}$ which means that newsvendor i 's share of transshipment profit of inventory from i to j is not too high; and (b) $c_{ji} - s_i \geq \frac{r_i - s_i}{2}$ which means that the net risk of newsvendor i buying inventory from j is not too low. Collectively, this means that newsvendor i does not dominate newsvendor j in the inventory transshipment in terms of profit allocation and transshipped amount. Furthermore, if the condition also holds for newsvendor j , then neither newsvendor would dominate the other and they should hold relatively fair positions in the game.

In the remainder of this paper, for technical tractability, we assume that $\pi_i^T(x_i, K - x_i)$ is quasi-concave in $0 \leq x_i \leq K$. For a given K , suppose $x_i^{M,K}$ maximizes $\pi_i^T(x_i, K - x_i)$ in $0 \leq x_i \leq K$. That is, if supply is fully allocated, the newsvendor i prefers that he receives $x_i^{M,K}$. We show that under certain conditions, $\Pi_i^{TL}(y_i, y_j)$ is quasi-concave in y_i .

Proposition 2 *Given supply capacity K , newsvendor i 's expected profit function $\Pi_i^{TL}(y_i, y_j)$ is quasi-concave in y_i for any given $0 \leq y_j \leq K$ under the following two conditions:*

- (1) $\pi_i^T(x_i, K - x_i)$ is quasi-concave in $0 \leq x_i \leq K$;
- (2) $\frac{r_i - c_{ji}}{c_{ij} - t_{ij} - s_i} \geq \frac{\Pr(D_i + D_j \leq K \leq x_i^{M,K} + D_j)}{\Pr(x_i^{M,K} + D_j \leq K \leq D_i + D_j)}$.

We offer two interpretations of Condition (2). First, notice that $r_i - c_{ji}$ is newsvendor i 's share of transshipment profit of each unit inventory from j to i , and $c_{ij} - t_{ij} - s_i$ is newsvendor i 's share of transshipment profit of each unit inventory from i to j . Then the condition means that the ratio between these two shares should be no less than a threshold. The threshold is the ratio of two probabilities at $x_i = x_i^{M,K}$, $x_j = K - x_i^{M,K}$. $\Pr(D_i + D_j \leq K \leq x_i^{M,K} + D_j)$ is the probability that newsvendor j has residual demand and this residual demand can be fully satisfied by surplus inventory from i to j ; and $\Pr(x_i^{M,K} + D_j \leq K \leq D_i + D_j)$ is the probability that newsvendor i has residual demand and all of newsvendor j 's surplus inventory is transshipped to i to (partially or fully) satisfy this residual demand.

Note that this ratio is not necessarily greater than 1, which means the condition is not requiring $r_i - c_{ji} \geq c_{ij} - t_{ij} - s_i$. Second, Condition (2) is equivalent to $x_i^{M,K} \leq R_i^T(K - x_i^{M,K})$. It means that, in response to newsvendor j 's order quantity $K - x_i^{M,K}$, newsvendor i 's best decision, $x_i^{M,K}$, becomes less than $R_i^T(K - x_i^{M,K})$ which is his best decision if the supply was unlimited. In this sense, the limited supply restricts newsvendor i 's best decision.

Conditions (1) and (2) in Proposition 2 are general conditions involving price parameters and their relation with demands of the newsvendors. While we observe that they usually satisfied in numerical experiments, we keep their general form for theoretical rigorousness. In the remainder of the paper, we assume that these two conditions hold.

Subsequently, newsvendor i 's best response can be characterized. For any y_j placed by the newsvendor j , the newsvendor i finds his optimal order by considering his best response $R_i^T(y_j)$ in the unlimited-supply case, his preferred allocation $x_i^{M,K}$ and the order limit K . Specifically, if $R_i^T(y_j) + y_j \leq K$, then newsvendor i simply orders $R_i^T(y_j)$. If $R_i^T(y_j) + y_j > K$, he needs to compare $K - y_j$ and $x_i^{M,K}$. If $K - y_j \geq x_i^{M,K}$ which means that competing for more inventory only leads to less profit, then he orders $K - y_j$. If $K - y_j < x_i^{M,K}$, then he will increase his order to $\psi_i(y_j | x_i^{M,K})$ so that he can receive $x_i^{M,K}$, unless $x_i^{M,K} > \varphi_i(K, y_j)$ in which case he orders the largest quantity allowed, K . To summarize the analysis above, we have

$$R_i^{TL}(y_j) = \begin{cases} R_i^T(y_j), & \text{if } y_j + R_i^T(y_j) \leq K, \\ K - y_j, & \text{if } y_j + R_i^T(y_j) > K \text{ and } \\ & x_i^{M,K} + y_j \leq K, \\ \psi_i(y_j | x_i^{M,K}), & \text{if } y_j + R_i^T(y_j) > K \text{ and } \\ & K - y_j < x_i^{M,K} \leq \varphi_i(K, y_j), \\ K, & \text{if } y_j + R_i^T(y_j) > K \text{ and } \\ & x_i^{M,K} > \varphi_i(K, y_j). \end{cases}$$

Furthermore, by Proposition 2, there exists a Nash equilibrium of orders. To characterize this equilibrium $\mathbf{y}^{N, TL}$ and the resulting $\mathbf{x}^{N, TL}$, we need the next result. Denote $\bar{K} = x_1^{N, T} + x_2^{N, T}$ where $\mathbf{x}^{N, T}$ is the Nash equilibrium in Model NT.

Proposition 3 *Under the same conditions in Proposition 2, we have*

- (1) *There exists K_i^0 such that $x_i^{M,K} = K$ when $K \leq K_i^0$ and $x_i^{M,K} < K$ is non-increasing in K when $K > K_i^0$, $i = 1, 2$;*
- (2) *There exists $\underline{K}$ such that $\underline{K} = x_1^{M,K} + x_2^{M,K}$;*
- (3) $\underline{K} \leq \bar{K}$.

Result (1) in Proposition 3 implies that when the supply capacity is relatively small (no more than a threshold K_i^0), newsvendor i wishes to take all available supply because it is highly unlikely that newsvendor j will have any surplus for transshipment and newsvendor i should possess inventory as early as possible. When supply capacity is relatively

TABLE 2 Equilibrium orders and inventory allocation in Model NTL

Case	$\mathbf{y}^{N,TL}$	$\mathbf{x}^{N,TL}$
$K \geq \bar{K}$	$(x_1^{N,T}, x_2^{N,T})$	$(x_1^{N,T}, x_2^{N,T})$
$\underline{K} < K < \bar{K}$	$\max\{x_1^{M,K}, K - x_2^{I,K}\} \leq y_1^{N,TL} \leq \min\{K - x_2^{M,K}, x_1^{I,K}\},$ $y_2^{N,TL} = K - y_1^{N,TL}$	$(y_1^{N,TL}, y_2^{N,TL})$
$K = \underline{K}$	$(y_1^{N,TL}, \psi_2(y_1^{N,TL} x_2^{M,K})), x_1^{M,K} \leq y_1^{N,TL} \leq K$	$(x_1^{M,K}, x_2^{M,K})$
$K < \underline{K}$ $x_1^{M,K} < \frac{K}{2}$	$(\psi_1(K x_1^{M,K}), K)$	$(x_1^{M,K}, K - x_1^{M,K})$
$x_2^{M,K} < \frac{K}{2}$	$(K, \psi_2(K x_2^{M,K}))$	$(K - x_2^{M,K}, x_2^{M,K})$
$\min\{x_1^{M,K}, x_2^{M,K}\} \geq \frac{K}{2}$	(K, K)	$(\frac{K}{2}, \frac{K}{2})$

TABLE 3 Equilibrium analysis of Example 1

K	$\mathbf{x}^{M,K}$	$\mathbf{x}^{I,K}$	$\mathbf{x}^{N,T}$	Case	$\mathbf{y}^{N,TL}$	$\mathbf{x}^{N,TL}$	Unique?
9	(6.09, 3.82)	(6.42, 4.20)	(6.19, 3.97)	$K < \underline{K}$	(9.00, 6.64)	(5.18, 3.82)	Yes
10	(5.80, 1.33)	(6.23, 4.00)	(6.19, 3.97)	$\underline{K} < K < \bar{K}$	$6.00 \leq y_1^{N,TL} \leq 6.23$ $y_2^{N,TL} = 10 - y_1^{N,TL}$	$(y_1^{N,TL}, y_2^{N,TL})$	No
12	(5.06, 0.00)	(5.75, 3.27)	(6.19, 3.97)	$K \geq \bar{K}$	(6.20, 3.96)	(6.19, 3.97)	Yes

large (more than K_i^0), newsvendor i prefers to push more initial inventory, and thus overstocking risk, to newsvendor j and uses inventory transshipment to satisfy newsvendor i 's own demand after demand is realized.

Now we derive the Nash equilibrium of orders. When K is very small, each newsvendor wants all supply and we have $x_1^{M,K} = x_2^{M,K} = K$. Therefore, $\mathbf{y}^{N,TL} = (K, K)$ is the unique Nash equilibrium and $\mathbf{x}^{N,TL} = (\frac{K}{2}, \frac{K}{2})$. As long as $K < \underline{K}$, both newsvendors inflate their orders with aim to receive their desired inventories. Specifically, if $x_i^{M,K} < \frac{K}{2}$, then $y_i^{N,TL} = \psi_i(K | x_i^{M,K})$ and $x_i^{N,TL} = x_i^{M,K}$; if $x_i^{M,K} > \frac{K}{2}$, then $y_i^{N,TL} = K$ and $x_i^{N,TL} = K - x_j^{M,K}$. Both $\mathbf{y}^{N,TL}$ and $\mathbf{x}^{N,TL}$ are unique. If $K = \underline{K}$, any $\mathbf{y}^{N,TL}$ such that $x_i^{M,K} \leq y_i^{N,TL} \leq K$ and $y_j^{N,TL} = \psi_j(y_i^{N,TL} | x_j^{M,K})$ is a equilibrium and they all correspond to the same $\mathbf{x}^{N,TL} = (x_1^{M,K}, x_2^{M,K})$. As K increases to the range of $\underline{K} < K < \bar{K}$, best responses of the newsvendors overlap which lead to infinitely many Nash equilibria $\mathbf{y}^{N,TL}$ with $\max\{x_i^{M,K}, K - x_j^{I,K}\} \leq y_i^{N,TL} \leq \min\{K - x_j^{M,K}, x_i^{I,K}\}$ and $y_j^{N,TL} = K - y_i^{N,TL}$, where $x_i^{I,K}$ satisfies $x_i^{I,K} = R_i^T(K - x_i^{I,K})$ representing the intersection of $R_i^T(x_j)$ and $x_1 + x_2 = K$. In this case $\mathbf{x}^{N,TL} = \mathbf{y}^{N,TL}$. Finally, when $K \geq \bar{K}$, the supply is sufficient for both newsvendors. Then the unique Nash equilibrium of orders is $\mathbf{y}^{N,TL} = \mathbf{x}^{N,T}$ with inventories $\mathbf{x}^{N,TL} = \mathbf{y}^{N,TL}$. The above analysis is summarized by the next theorem and Table 2.

Theorem 1 Under the conditions in Proposition 2, Nash equilibrium of orders exists. There exists a unique Nash equilibrium if and only if $K < \underline{K}$ or $K \geq \bar{K}$. There exist infinitely many Nash equilibria if and only if $\underline{K} \leq K < \bar{K}$.

From Table 2 we see that a newsvendor tends to inflate his order quantity with the goal of receiving his optimal share

of the supply capacity, namely $x_i^{M,K}$. The newsvendor with the smaller optimal share ($x_i^{M,K} < \frac{K}{2}$) is favored because his inflated order quantity is less likely to be restricted by the order limit K . Moreover, the equilibrium inventory allocation $\mathbf{x}^{N,TL}$ is unique unless $\underline{K} < K < \bar{K}$. The uniqueness/non-uniqueness of Nash equilibrium indicates the relative significance of the gaming effects caused by limited supply and inventory transshipment. When the supply capacity is low ($K < \underline{K}$), both newsvendors have a high priority to secure inventory because there may not be enough surplus inventory for transshipment. Consequently, they tend to inflate their orders the same way as in Model NL. When the supply capacity is high ($K \geq \bar{K}$), competition for the limited inventory is not highly necessary. The newsvendors may safely order the unique Nash equilibrium $\mathbf{x}^{N,T}$ in Model NT. When $\underline{K} \leq K < \bar{K}$, neither the competition for limited supply nor the risk-pooling via inventory transshipment can dominate the newsvendors' decision-making, and consequently, infinitely many Nash equilibria exist.

We close this section by the following illustrative example.

Example 1 Let $w_1 = w_2 = 4$, $r_1 = r_2 = 6$, $s_1 = s_2 = 1$, $t_{12} = t_{21} = 2.5$, $c_{12} = c_{21} = 4$. The demands of the newsvendors are mutually independent, both uniformly distributed with $D_1 \sim U[4, 10]$, $D_2 \sim U[0, 10]$. Assume the proportional allocation rule is used. Table 3 presents the analysis of three capacity levels $K = 9, 10, 12$. The best responses of newsvendors in Model NT and NTL are plotted in Figures 2 to 5. Nash equilibrium is also marked. We see the order inflation phenomenon when $K = 9$. When $K = 10$ multiple Nash equilibria exist, which is not possible when the supply is unlimited.

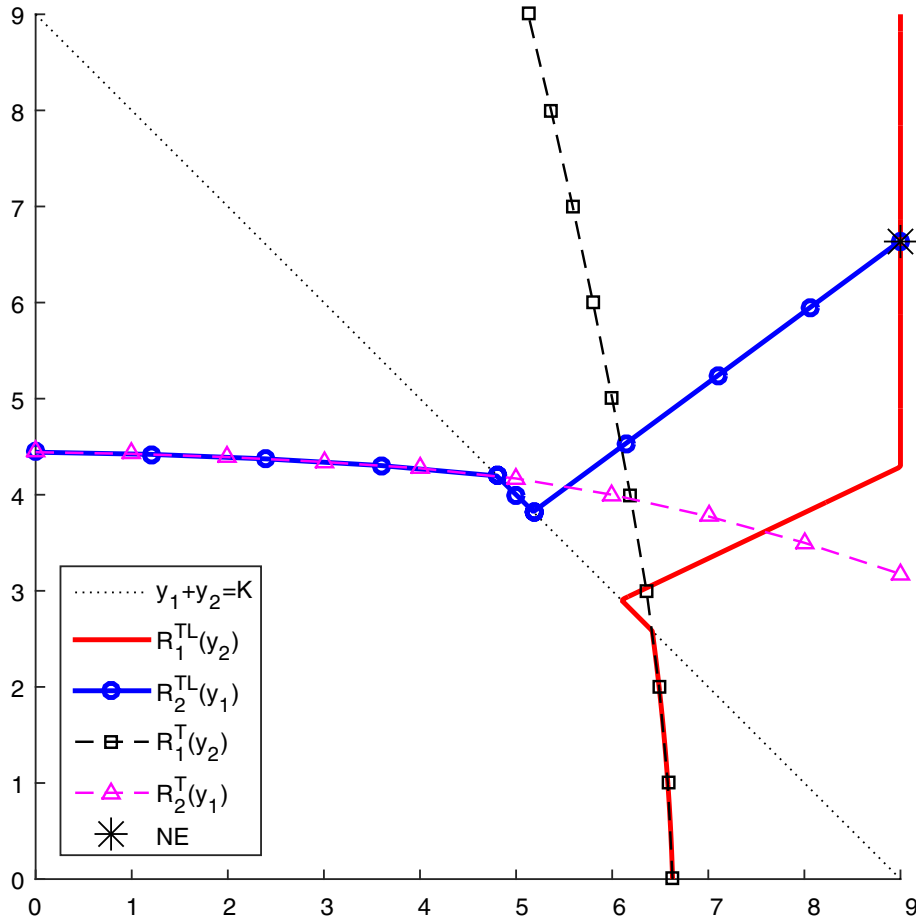


FIGURE 2 Example 1: $K = 9$ [Colour figure can be viewed at wileyonlinelibrary.com]

When $K = 12$, the supply is sufficient for the newsvendors to place the Nash equilibrium in Model NT.

5 | BENEFIT OF INVENTORY TRANSSHIPMENT

Undoubtedly, inventory transshipment always benefits both newsvendors when the supply is unlimited. Intuitively, transshipment provides a second chance for each newsvendor to continue satisfying his demand and therefore, to generate more profit. To see this, one can compare the Model NT and the classic newsvendor model and note that $\pi_i^T(x_i^{N,T}, x_j^{N,T}) \geq \pi_i^T(x_i^*, x_j^*)$. In this section, we investigate whether this remains true under the limited supply by comparing Model NTL and the baseline Model NL. Throughout this section we assume that the Nash equilibrium of orders in Model NTL is unique, which means $K < \underline{K}$ or $K \geq \bar{K}$. The results remain valid for multiple Nash equilibria case (see Appendix E).

If the newsvendor i has a higher profit at the Nash equilibrium of Model NTL than that of Model NL, that is, $\Pi_i^{TL}(y_i^{N,TL}, y_j^{N,TL}) \geq \Pi_i^L(y_i^{N,L}, y_j^{N,L})$, he is considered to be better off and in this sense, inventory transshipment benefits the newsvendor i . In Model NTL, there are four possible

cases of the allocated inventory $x_i^{N,TL}$ when Nash equilibrium is unique: $x_i^{N,TL} = \frac{K}{2}$ when both newsvendors order K ; $x_i^{N,TL} = x_i^{M,K}$ when the newsvendor i receives his preferred allocation; $x_i^{N,TL} = K - x_j^{M,K}$ when the other newsvendor j receives preferred allocation; and $x_i^{N,TL} = x_i^{N,T}$ when the supply is sufficient to satisfy the equilibrium orders of Model NT. In Model NL, there are three cases of $x_i^{N,L}$: $x_i^{N,L} = \frac{K}{2}$, $x_i^{N,L} = K - x_j^*$ and $x_i^{N,L} = x_i^*$. The next theorem characterizes the scenarios when the newsvendor i is better off.

Theorem 2 $\Pi_i^{TL}(y_i^{N,TL}, y_j^{N,TL}) \geq \Pi_i^L(y_i^{N,L}, y_j^{N,L})$ if

- i $x_i^{N,TL} = x_i^{N,T}$, or
- ii $x_i^{N,TL} = x_i^{M,K}$, or
- iii $x_i^{N,TL} \geq x_i^{N,L}$.

Theorem 2 allows the newsvendor i to check his equilibrium inventory allocation $x_i^{N,TL}$ in order to assess the benefit of inventory transshipment. Scenario (i) occurs when the supply capacity is large enough for the retailers to receive $\mathbf{x}^{N,T}$ of Model NT. Similar to the unlimited supply case, the newsvendor i becomes better off. Scenario (ii) implies that the limited supply is allocated such that the newsvendor i maximizes his own profit in Model NTL, which makes him better off.

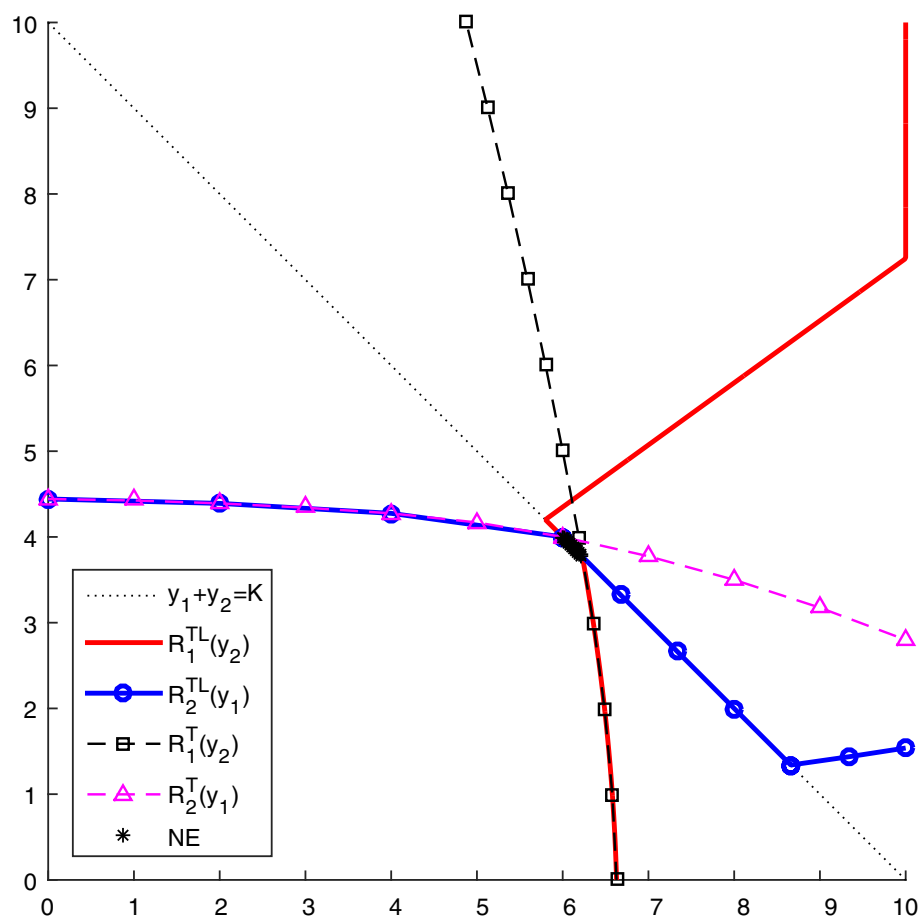


FIGURE 3 Example 1: $K = 10$ [Colour figure can be viewed at wileyonlinelibrary.com]

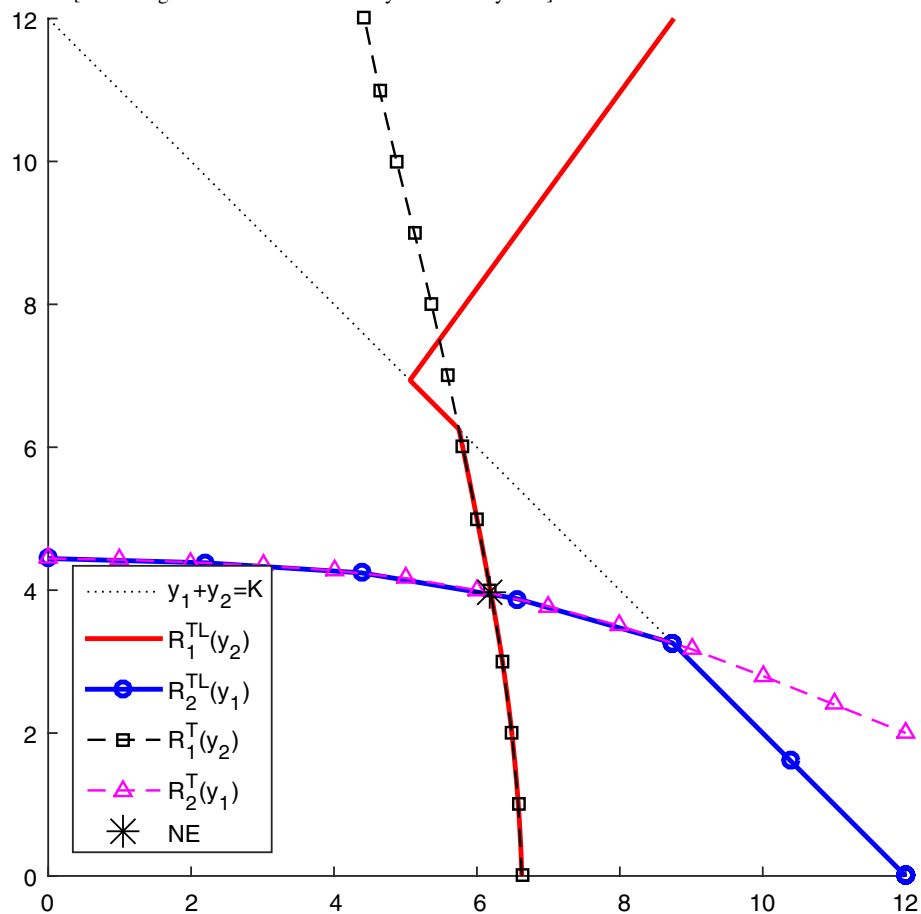


FIGURE 4 Example 1: $K = 12$ [Colour figure can be viewed at wileyonlinelibrary.com]

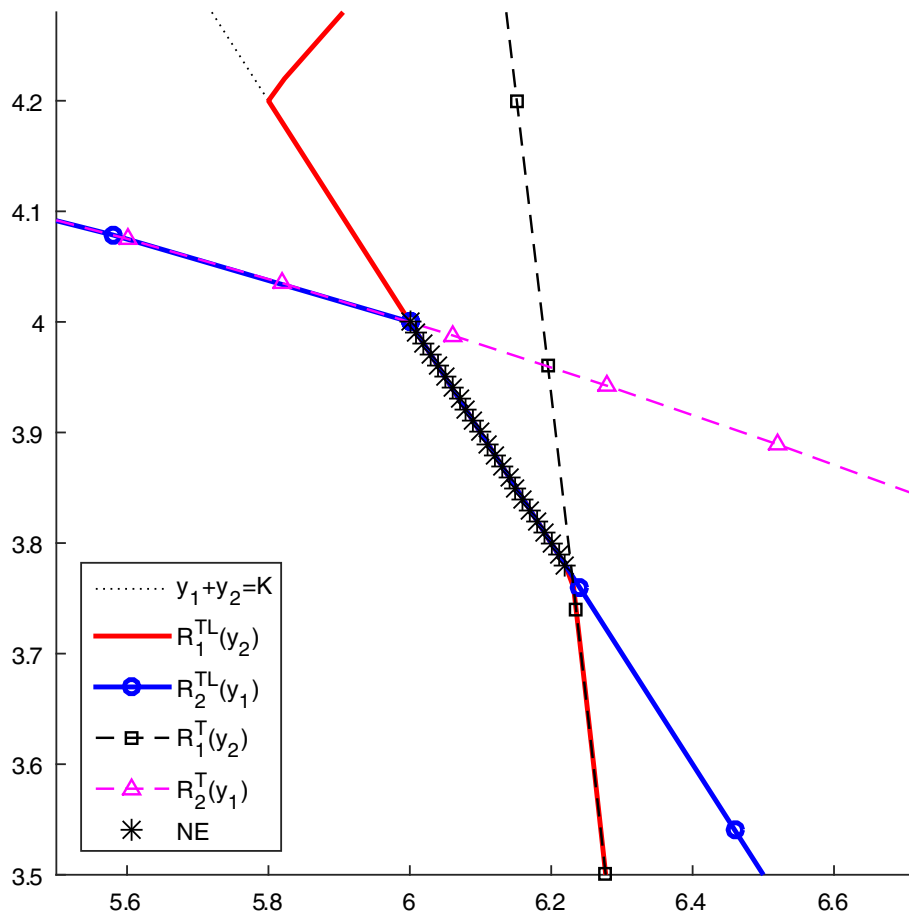


FIGURE 5 Figure 3 zoomed in [Colour figure can be viewed at wileyonlinelibrary.com]

Scenario (iii) refers to two cases of equilibrium inventories: $x_i^{N,TL} = x_j^{N,TL} = \frac{K}{2}$ or $x_i^{N,TL} = K - x_j^{M,K}$, $x_j^{N,TL} = x_j^{M,K}$. Intuitively, if the newsvendor i receives no less inventory allocation than in Model NL, that is, $x_i^{N,TL} \geq x_i^{N,L}$, he may have a higher chance of meeting his own demand and surplus inventory for transshipment and consequently, be better off. In these three scenarios, the newsvendor i prefers to participate in inventory transshipment. However, Theorem 2 leaves out the cases $x_i^{N,TL} = x_j^{N,TL} = \frac{K}{2} < x_i^{N,L}$ and $x_i^{N,TL} = K - x_j^{M,K} < x_i^{N,L}$, $x_j^{N,TL} = x_j^{M,K}$. For these cases, further analysis is necessary before the newsvendor i can assess whether inventory transshipment is beneficial. Note that to directly formulate the cases in Theorem 2 with model parameters can be extremely complex because $\mathbf{x}^{N,TL}$, $\mathbf{x}^{N,T}$ and $\mathbf{x}^{N,L}$ involve the model parameters in a collective manner (first-order conditions of profit functions). Actually, a less complex task of comparing $x_i^{N,T}$ and x_i^* demands tedious analysis (Rudi et al., 2001).

Inventory transshipment requires at least two participants. Knowing whether and when both newsvendors will benefit from inventory transshipment in Model NTL may lead to better mechanism design that motivates them to participate. Following Theorem 2, we have the next two results. Moreover, Table 4 shows which newsvendor is guaranteed to be better off in any given scenario of equilibrium inventory $\mathbf{x}^{N,TL}$ of Model NTL and $\mathbf{x}^{N,L}$ of Model NL.

Proposition 4 *At least one newsvendor is better off in Model NTL than in Model NL.*

Proposition 5 *Both newsvendors are better off in Model NTL than in Model NL if*

- (i) $\mathbf{x}^{N,TL} = \mathbf{x}^{N,T}$, or
- (ii) $x_i^{N,TL} = x_i^{M,K}$ and $x_j^{N,TL} = K - x_i^{M,K} \geq x_j^{N,L}$ for $i = 1, j = 2$ or $i = 2, j = 1$, or
- (iii) $x_i^{N,TL} = \frac{K}{2} \geq x_i^{N,L}$ for $i = 1, 2$.

We see that inventory transshipment only can benefit one newsvendor for sure. Proposition 5 implies that both newsvendors can benefit when (i) the supply is large enough, or (ii) one newsvendor receives his preferred allocation and the other newsvendor gets more inventory than in Model NL, or (iii) the supply is scarce and both newsvendors receive no less than in Model NL. Otherwise, one newsvendor could become worse off with inventory transshipment under limited supply, as illustrated by the following example.

Example 2 Consider an instance with $w_1 = w_2 = 9$, $r_1 = 10.5$, $r_2 = 10$, $s_1 = s_2 = 1$, $t_{12} = 5$, $t_{21} = 3$, $c_{12} = 7$, $c_{21} = 9$, $D_1 \sim U[4, 18]$, $D_2 \sim U[2, 9]$ and $K = 8$. In Model NL, $x_1^* = 6.21$ and $x_2^* = 2.78$. Therefore, $\mathbf{y}^{N,L} = (8, 4.26)$ and $\mathbf{x}^{N,L} = (5.22, 2.78)$

TABLE 4 Newsvendor(s) guaranteed to be better off in Model NTL than in Model NL

	$\mathbf{x}^{N,L}$			
	$\left(\frac{K}{2}, \frac{K}{2}\right)$	$(x_1^*, K - x_1^*)$	$(K - x_2^*, x_2^*)$	(x_1^*, x_2^*)
$\mathbf{x}^{N,TL}$	$\left(\frac{K}{2}, \frac{K}{2}\right)$	1,2	1	2
	$(x_1^{M,K}, K - x_1^{M,K})$	1	1	1
	$(K - x_2^{M,K}, x_2^{M,K})$	2	2	2
	$(x_1^{N,T}, x_2^{N,T})$	1,2	1,2	1,2

TABLE 5 Equilibrium profits in Example 2 as transshipment prices vary

δ	c_{12}	c_{21}	Model NL			Model NTL		
			Π_1^L	Π_2^L	Π_t^L	Π_1^{TL}	Π_2^{TL}	Π_t^{TL}
0%	6.00	10.50	7.33	2.39	9.72	6.00	3.20	9.20
10%	6.40	9.85	7.33	2.39	9.72	6.18	3.02	9.20
20%	6.80	9.20	7.33	2.39	9.72	6.68	2.86	9.54
30%	7.20	8.55	7.33	2.39	9.72	7.09	2.76	9.85
40%	7.60	7.90	7.33	2.39	9.72	7.29	2.69	9.98
50%	8.00	7.25	7.33	2.39	9.72	7.41	2.63	10.04
60%	8.40	6.60	7.33	2.39	9.72	7.50	2.57	10.07
70%	8.80	5.95	7.33	2.39	9.72	7.58	2.52	10.10
80%	9.20	5.30	7.33	2.39	9.72	7.64	2.48	10.12
90%	9.60	4.65	7.33	2.39	9.72	7.70	2.43	10.13
100%	10.00	4.00	7.33	2.39	9.72	7.75	2.39	10.14

which results in $\Pi_1^L(y_1^{N,L}, y_2^{N,L}) = 7.33$ and $\Pi_2^L(y_2^{N,L}, y_1^{N,L}) = 2.39$. In Model NTL, we have $x_1^{M,K} = 6.47$ and $x_2^{M,K} = 3.56$. Then $\mathbf{y}^{N,TL} = (8, 6.42)$ and $\mathbf{x}^{N,TL} = (4.44, 3.56)$. The profits are $\Pi_1^{TL}(y_1^{N,TL}, y_2^{N,TL}) = 6.84$ and $\Pi_2^{TL}(y_2^{N,TL}, y_1^{N,TL}) = 2.82$. Therefore, $\Pi_1^{TL}(y_1^{N,TL}, y_2^{N,TL}) < \Pi_1^L(y_1^{N,L}, y_2^{N,L})$ which means that the newsvendor 1 is worse off. The newsvendor 2 is indeed better off as implied by Theorem 2 and Proposition 4. In this example $x_1^{N,TL} = K - x_2^{M,K} < x_1^{N,L}$ and $x_2^{N,TL} = x_2^{M,K}$ which is not any of the cases in Proposition 5.

To understand the reason why the newsvendor 1 becomes worse off, we notice that $c_{21} - s_2 - t_{21} = 5 > r_1 - c_{21} = 1$ and $r_2 - c_{12} = 3 > c_{12} - s_1 - t_{12} = 1$. Consequently, the newsvendor 1 receives a smaller share of the extra profit generated by inventory transshipment than the newsvendor 2 does. Unfortunately, his profit loss $\pi_1(x_1^{N,L}) - \pi_1(x_1^{N,TL})$ is not fully compensated by the shared profit from inventory transshipment. Therefore, the newsvendor 1 becomes worse off.

Total profit of the two newsvendors is an important indicator of the supply chain performance because higher total profit indicates higher service level. When the supply is unlimited, inventory transshipment always increases the total newsvendor profit. However, in the limited supply case, as we observe in Example 2, the total newsvendor

profit in Model NTL ($\Pi_t^{TL}(\mathbf{y}^{N,TL}) = \Pi_1^{TL}(y_1^{N,TL}, y_2^{N,TL}) + \Pi_2^{TL}(y_2^{N,TL}, y_1^{N,TL}) = 9.66$) is lower than that in Model NL ($\Pi_t^L(\mathbf{y}^{N,L}) = \Pi_1^L(y_1^{N,L}, y_2^{N,L}) + \Pi_2^L(y_2^{N,L}, y_1^{N,L}) = 9.72$). It indicates that the limited inventory supply is not allocated in the most effective way. Therefore, it is highly necessary to design certain mechanisms so that inventory transshipment benefits the total newsvendor profit when the supply is limited. Transshipment prices can be the key of such mechanisms.

To verify the insight above and investigate how the transshipment prices make a newsvendor better or worse off, we conduct a numerical study based on Example 2. We let c_{12} and c_{21} vary while keeping $\frac{c_{12} - s_1 - t_{12}}{r_2 - s_1 - t_{12}} = \frac{r_1 - c_{21}}{r_1 - s_2 - t_{21}} = \delta$, where $\delta \in [0, 1]$ is the portion of transshipment profit that belongs to the newsvendor 1. For $\delta = 0\%, 10\%, \dots, 100\%$, we compute $\Pi_1^{TL}(y_1^{N,TL}, y_2^{N,TL})$, $\Pi_2^{TL}(y_2^{N,TL}, y_1^{N,TL})$ and $\Pi_t^{TL}(\mathbf{y}^{N,TL})$ to compare with $\Pi_1^L(y_1^{N,L}, y_2^{N,L})$, $\Pi_2^L(y_2^{N,L}, y_1^{N,L})$ and $\Pi_t^L(\mathbf{y}^{N,L})$, respectively. Results are shown in Table 5.

We observe that $\Pi_1^{TL}(y_1^{N,TL}, y_2^{N,TL})$ increases and $\Pi_2^{TL}(y_2^{N,TL}, y_1^{N,TL})$ decreases as δ becomes higher. The newsvendor 1 is worse off when $\delta \leq 40\%$ and the total newsvendor profit is lower than that in Model NL when $\delta \leq 20\%$. This implies that transshipment prices affect newsvendors' equilibrium profits by determining how the transshipment profit is allocated. Therefore, transshipment prices, if carefully chosen, may ensure the benefit of inventory transshipment for both newsvendors and eventually, their incentives to participate under the supply constraint.

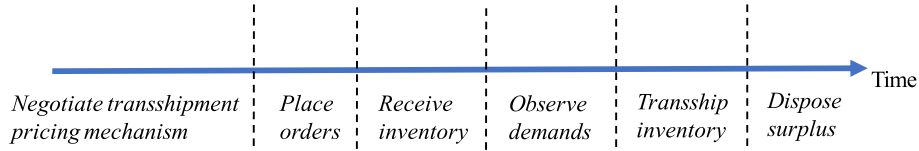


FIGURE 6 Sequence of events in Model NTL with proposed coordinating mechanism [Colour figure can be viewed at wileyonlinelibrary.com]

6 | A COORDINATING MECHANISM

In this section, we provide a mechanism such that at the Nash equilibrium $\mathbf{y}^{N, TL}$ of Model NTL, both newsvendors are better off than in Model NL, and total newsvendor profit is maximized. In this sense, this mechanism will coordinate the newsvendors. As the analysis in Section 5 suggests, the transshipment prices should play an important role. However, Hu et al. (2007) have shown that there may not always exist a pair of fixed transshipment prices that can coordinate the newsvendors in Model NT. Therefore, in this section we consider that the transshipment prices c_{12} and c_{21} vary with the orders $\mathbf{y}$. This idea is also seen in an implicit pricing coordinating contract (Hezarkhani and Kubiak, 2010). By adopting their key design, we derive a coordinating mechanism for Model NTL that allows newsvendors to negotiate on transshipment prices before they place orders.

Formally, the proposed coordinating mechanism (CM) runs as the following two stages with a sequence of events shown in Figure 6:

Stage 1: Before placing orders, the newsvendors negotiate and agree on a transshipment pricing mechanism $(c_{12}(\mathbf{y}), c_{21}(\mathbf{y}))$ as a function of any feasible order quantity $\mathbf{y}$.

Stage 2: After the newsvendors place orders $\mathbf{y}$, they use $(c_{12}(\mathbf{y}), c_{21}(\mathbf{y}))$ as transshipment prices.

The negotiated transshipment prices $c_{12}(\mathbf{y})$ and $c_{21}(\mathbf{y})$ can be determined by the general Nash bargaining solution (GNBS) to the newsvendors' bargaining problem. In general, a bargaining problem consists of a set of *utility* pairs of both players and a *disagreement point* representing the utilities if the players fail to reach an agreement. In Model NTL, associated with any given (y_1, y_2) , the utilities of the newsvendors are their expected profits $((\Pi_1^{TL}(y_1, y_2; \mathbf{c}), \Pi_2^{TL}(y_2, y_1; \mathbf{c}))$ for given $\mathbf{c} = (c_{12}, c_{21})$. The disagreement point is $(\Pi_1^L(y_1^{N,L}, y_2^{N,L}), \Pi_2^L(y_2^{N,L}, y_1^{N,L}))$ because if the negotiation fails, there will be no inventory transshipment and the game becomes Model NL.

According to the general Nash bargaining solution (Nash, 1951; Roth, 1979), for any given orders (y_1, y_2) , the negotiated transshipment prices $(c_{12}(\mathbf{y}), c_{21}(\mathbf{y}))$ form an optimal solution of the following optimization problem (P_y) in $\mathbf{c}$:

$$\max_{\mathbf{c}} f_y(\mathbf{c}) := [\Pi_1^{TL}(y_1, y_2; \mathbf{c}) - \Pi_1^L(y_1^{N,L}, y_2^{N,L})]^{\alpha_1} \times [\Pi_2^{TL}(y_2, y_1; \mathbf{c}) - \Pi_2^L(y_2^{N,L}, y_1^{N,L})]^{\alpha_2}$$

$$\text{s.t. } \Pi_1^{TL}(y_1, y_2; \mathbf{c}) \geq \Pi_1^L(y_1^{N,L}, y_2^{N,L}), \quad (1)$$

$$\Pi_2^{TL}(y_2, y_1; \mathbf{c}) \geq \Pi_2^L(y_2^{N,L}, y_1^{N,L}), \quad (2)$$

where $\alpha_1, \alpha_2 \in (0, 1)$ are the newsvendors' bargaining powers with $\alpha_1 + \alpha_2 = 1$.

Note that the total newsvendor profit $\Pi_t^{TL}(\mathbf{y})$ does not depend on transshipment prices $\mathbf{c}$. If $\Pi_t^{TL}(\mathbf{y}) < \Pi_t^L(\mathbf{y}^{N,L})$, then constraints (1) and (2) cannot be simultaneously satisfied. Consequently, the negotiation will fail. Hence, we only consider (y_1, y_2) such that $\Pi_t^{TL}(\mathbf{y}) \geq \Pi_t^L(\mathbf{y}^{N,L})$. It is easy to check that $f_y(\mathbf{c})$ is concave in $\mathbf{c}$ by noting $\Pi_1^{TL}(y_1, y_2; \mathbf{c})$ is a linear function in $\mathbf{c}$ (proof in Hezarkhani and Kubiak, 2010). Then $\mathbf{c}$ globally maximizes $f_y(\mathbf{c})$ if and only if $\mathbf{c}$ satisfies the first order conditions $\frac{\partial f_y(\mathbf{c})}{\partial c_{12}} = 0$ and $\frac{\partial f_y(\mathbf{c})}{\partial c_{21}} = 0$, both of which are equivalent to

$$\begin{aligned} & T_{12}(\mathbf{y})c_{12} - T_{21}(\mathbf{y})c_{21} \\ &= \alpha_1[r_2T_{12}(\mathbf{y}) - (t_{21} + s_2)T_{21}(\mathbf{y}) + \Pi_2^L(y_2, y_1) - \Pi_2^L(y_2^{N,L}, y_1^{N,L})] \\ & - \alpha_2[r_1T_{21}(\mathbf{y}) - (t_{12} + s_1)T_{12}(\mathbf{y}) + \Pi_1^L(y_1, y_2) - \Pi_1^L(y_1^{N,L}, y_2^{N,L})] \end{aligned} \quad (3)$$

where $T_{ji}(\mathbf{y}) = \mathbf{E}[\min\{(D_i - \varphi_i(y_i, y_j))^+, (\varphi_j(y_j, y_i) - D_j)^+\}]$ denotes the expected amount of inventory transshipped from newsvendor j to newsvendor i , $i, j = 1, 2, i \neq j$. To better understand Equation (3), we reorganize it as

$$\frac{\Pi_1^{TL}(y_1, y_2; \mathbf{c}(\mathbf{y})) - \Pi_1^L(y_1^{N,L}, y_2^{N,L})}{\Pi_2^{TL}(y_2, y_1; \mathbf{c}(\mathbf{y})) - \Pi_2^L(y_2^{N,L}, y_1^{N,L})} = \frac{\alpha_1}{\alpha_2}.$$

This means that each newsvendor receives expected extra profit from inventory transshipment in proportion to his bargaining power. For example, when $\alpha_1 = \alpha_2$, both newsvendors' expected transshipment profits are equal. More precisely, the relation between each newsvendor's profit and their total profit is shown in the next result.

Proposition 6 For any $\mathbf{y}$ such that $\Pi_t^{TL}(\mathbf{y}) \geq \Pi_t^L(\mathbf{y}^{N,L})$, if the transshipment prices $\mathbf{c}(\mathbf{y})$ are determined by the generalized Nash bargaining solution, then

$$\begin{aligned} \Pi_i^{TL}(y_i, y_j; \mathbf{c}(\mathbf{y})) &= \Pi_i^L(y_i^{N,L}, y_j^{N,L}) + \alpha_i[\Pi_t^{TL}(\mathbf{y}) - \Pi_t^L(\mathbf{y}^{N,L})] \\ &\text{for } i, j = 1, 2, i \neq j. \end{aligned}$$

Proposition 6 means that each newsvendor's profit is his equilibrium profit in Model NL plus a fixed portion, specified by his bargaining power, of the total extra profit generated by transshipment. In this way, each newsvendor's own profit is aligned with their total profit. Therefore, the Nash equilibrium, at which each newsvendor maximizes his own profit,

also maximizes the total newsvendor profit. Formally, we have the next theorem.

Theorem 3 *If the newsvendors may negotiate the transshipment prices before placing orders as the mechanism CM specifies, then the equilibrium orders $\mathbf{y}^{N,TL}$ maximize their total profit $\Pi_i^{TL}(\mathbf{y})$ with both newsvendors better off.*

This coordinating mechanism can be a potential solution when either newsvendor finds that he does not benefit from inventory transshipment in Model NTL. Rather than seeing the inventory transshipment fail, the newsvendors may seek to negotiate the transshipment prices and achieve a win-win result. They can estimate the costs associated with the negotiation process to assess whether the profit added is worth the effort.

We note that there can be multiple optimal solutions to problem (P_y) and by the model settings the transshipment prices $\mathbf{c}(\mathbf{y})$ should satisfy $t_{ij} + s_i \leq c_{ij}(\mathbf{y}) \leq \min\{r_i, r_j\}$ for $i, j = 1, 2, i \neq j$. Therefore, the retailers should choose an optimal solution that meets the practical requirements of inventory transshipment when there is one. We illustrate how this mechanism can be implemented by the next example.

Example 3 Reconsider Example 2. Assume that the bargaining powers are $\alpha_1 = 0.6$, $\alpha_2 = 0.4$. For illustration purpose, we assume that the newsvendors only place integer orders. Table 6 shows $c_{12}(\mathbf{y})$ and $c_{21}(\mathbf{y})$ for each $\mathbf{y} = (y_1, y_2)$, $y_1, y_2 = 0, 1, \dots, K$. The symbol “-” indicates that $\Pi_i^{TL}(\mathbf{y}) < \Pi_i^{N,L}(\mathbf{y})$ in which case no agreement will be reached and the newsvendors choose not to transship inventory. Note that, besides Equation (3), $\mathbf{c}(\mathbf{y})$ should also satisfy $t_{ij} + s_i \leq c_{ij}(\mathbf{y}) \leq \min\{r_i, r_j\}$ for $i, j = 1, 2, i \neq j$. All of such $\mathbf{c}(\mathbf{y})$ form a line segment when $T_{12}(\mathbf{y}) \neq 0$ or $T_{21}(\mathbf{y}) \neq 0$. We choose the center of the segment. If $T_{12}(\mathbf{y}) = 0$ and $T_{21}(\mathbf{y}) = 0$, then any $\mathbf{c}(\mathbf{y})$ such that $t_{12} + s_1 \leq c_{12}(\mathbf{y}) \leq \min\{r_1, r_2\}$ and $t_{21} + s_2 \leq c_{21}(\mathbf{y}) \leq \min\{r_1, r_2\}$ can be the negotiated transshipment prices, and we choose its center $(c_{12}(\mathbf{y}), c_{21}(\mathbf{y})) = \left(\frac{t_{12} + s_1 + \min\{r_1, r_2\}}{2}, \frac{t_{21} + s_2 + \min\{r_1, r_2\}}{2} \right)$. Table 7 shows the profits $\Pi_1^{TL}(y_1, y_2; \mathbf{c}(\mathbf{y}))$ and $\Pi_2^{TL}(y_2, y_1; \mathbf{c}(\mathbf{y}))$ with the negotiated transshipment prices. For (y_1, y_2) such that $\Pi_i^{TL}(\mathbf{y}) < \Pi_i^{N,L}(\mathbf{y})$, we set $\Pi_1^{TL}(y_1, y_2, \mathbf{c}(\mathbf{y})) = \Pi_1^{N,L}(y_1^{N,L}, y_2^{N,L})$ and $\Pi_2^{TL}(y_2, y_1, \mathbf{c}(\mathbf{y})) = \Pi_2^{N,L}(y_2^{N,L}, y_1^{N,L})$. The red boxes highlight the newsvendor 1’s best profits given the newsvendor 2’s order, and the blues ones for the newsvendor 2’s. We observe that (7, 3) is a Nash equilibrium that maximizes $\Pi_1^{TL}(y_1, y_2; \mathbf{c}(\mathbf{y}))$,

$\Pi_2^{TL}(y_2, y_1; \mathbf{c}(\mathbf{y}))$ and $\Pi_i^{TL}(\mathbf{y})$. Therefore, the newsvendors should order $y_1 = 7, y_2 = 3$ and use $c_{12} = 6.72, c_{21} = 7.00$.

7 | CONCLUDING REMARKS

In this paper, we have studied the inventory transshipment game of two retailers with limited total supply. We single out their ordering decisions by assuming stochastic demands, retail prices and predetermined transshipment prices to be exogenous, and thus show the impact of limited supply capacity on each retailer’s inventory stocking strategies. We have obtained results for asymmetric retailers, general demand distributions and a broad class of allocation rules. Albeit the common difficulty of having simple and closed-form results for transshipment problems (sometimes even for simplified models such as identical retailers, price-dependent demand, etc.), we derive each retailer’s optimal order quantity as a best response to the other retailer’s order. The analysis clearly shows that the supply capacity significantly complicates retailers’ decisions for it may induce an inflation of each retailer’s order quantity while the inventory transshipment may reduce the needed inventory level. We have shown that a Nash equilibrium exists under certain conditions. The Nash equilibrium becomes unique if the gaming effect due to supply capacity is either much less or much more significant than that due to inventory transshipment. Otherwise, there may exist infinitely many Nash equilibria. All these results and conditions are in explicit forms that can be easily applied in practice as long as the specific allocation rule, demand distribution and cost structure of the retailers are known.

This research points out that the benefit of inventory transshipment should not be taken as granted. Although transshipment always benefits both retailers in the unlimited supply case (Model NT), we see that it is not always true when the supply capacity becomes limited. An important finding is that one of the retailers is guaranteed to be better off, while the other could be worse off compared with his equilibrium profit in the no-transshipment game (Model NL). Therefore, it is necessary that a retailer checks the conditions provided in this paper to evaluate his benefit before participating in inventory transshipment. Our numerical study indicates that the transshipment price is a critical factor in determining the allocation of the transshipment profit, and consequently, whether a retailer will be eventually benefited by inventory transshipment. A carefully chosen transshipment price may make both retailers better off. The supplier may need to offer the retailers certain types of incentive to promote inventory transshipment when her capacity is limited. As a possible solution for this, a coordinating mechanism allowing the retailers to negotiate the transshipment prices has been designed to achieve a win-win result.

TABLE 6 Negotiated transshipment prices in Example 3

y_1	y_2		1		2		3		4		5		6		7		8	
	c_{12}	c_{21}	c_{12}	c_{21}	c_{12}	c_{21}	c_{12}	c_{21}	c_{12}	c_{21}	c_{12}	c_{21}	c_{12}	c_{21}	c_{12}	c_{21}	c_{12}	c_{21}
0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	8.00	5.69	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	8.00	7.00	8.00	5.79	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	6.72	7.00	8.00	5.76	8.00	5.94	—	—	—	—	—	—
8	—	—	—	—	—	—	6.05	7.62	8.00	7.00	8.00	5.70	8.00	6.06	—	—	—	—

TABLE 7 Profits with negotiated transshipment prices in Example 3

y_2		1		2		3		4		5		6		7		8	
y_1	Π_1^{TL}	Π_2^{TL}	Π_1^{TL}	Π_2^{TL}	Π_1^{TL}	Π_2^{TL}	Π_1^{TL}	Π_2^{TL}	Π_1^{TL}	Π_2^{TL}	Π_1^{TL}	Π_2^{TL}	Π_1^{TL}	Π_2^{TL}	Π_1^{TL}	Π_2^{TL}	
0	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	
1	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	
2	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	
3	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	
4	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	
5	7.33	2.39	7.33	2.39	7.33	2.39	7.53	7.62	7.33	2.39	7.33	2.39	7.33	2.39	7.33	2.39	
6	7.33	2.39	7.33	2.39	7.33	2.39	7.60	2.52	7.46	2.48	7.33	2.39	7.33	2.39	7.33	2.39	
7	7.33	2.39	7.33	2.39	7.33	2.39	2.57	2.59	7.55	2.54	7.41	2.44	7.33	2.39	7.33	2.39	
8	7.33	2.39	7.33	2.39	7.33	2.39	7.61	2.58	7.60	2.57	7.51	2.51	7.36	2.41	7.33	2.39	

This work extends the scope of existing literature on inventory transshipment to the supply chains with limited supply. Our analysis provides a full prescription for a retailer's profit maximization decision on the quantity of ordering, participation of inventory transshipment, and negotiation of transshipment prices. The new insights into the effect of limited supply and the benefit of inventory transshipment may help supply chain owners to better manage a decentralized system of similar characteristics by designing a working coordinating mechanism. Among the first studies of its kind, this research provides a platform for modeling and analyzing more complex problems involving limited supply and inventory transshipment. Future research directions include extending the model in this paper to multiple retailers, considering other possible decisions of the retailer such as pricing and transshipment quantity, involving the supplier's decision problem on capacity investment and wholesale price, and more problem-specific analysis of inventory transshipment games with limited supply in business practices.

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APPENDIX A: PROOFS

Proof of Proposition 1 Define the following notations of probabilities (and derivatives) of different scenarios for given x_i, x_j :

$$\begin{aligned}\Delta_1 &= \Pr(x_j \leq D_j \leq x_i + x_j - D_i); \Delta_2 = \Pr(x_i + x_j - D_j \leq D_i \leq x_i); \Delta_3 = \Pr(x_i \leq D_i \leq x_i + x_j - D_j); \\ \Delta_4 &= \Pr(x_i + x_j - D_i \leq D_j \leq x_j); \Delta_5 = \Pr(x_i \geq D_i, x_j \geq D_j); \\ \delta_1 &= \Pr(D_i + D_j \leq x_i + x_j) f_{D_i|D_i+D_j \leq x_i+x_j}(x_i); \\ \delta_2 &= \Pr(D_i + D_j \geq x_i + x_j) f_{D_i|D_i+D_j \geq x_i+x_j}(x_i); \\ \delta_3 &= \Pr(D_i + D_j \leq x_i + x_j) f_{D_j|D_i+D_j \leq x_i+x_j}(x_j); \\ \delta_4 &= \Pr(D_i + D_j \geq x_i + x_j) f_{D_j|D_i+D_j \geq x_i+x_j}(x_j); \\ \delta_5 &= \Pr(D_i \leq x_i) f_{D_i+D_j|D_i \leq x_i}(x_i + x_j); \delta_6 = \Pr(D_j \leq x_j) f_{D_i+D_j|D_j \leq x_j}(x_i + x_j);\end{aligned}$$

Then we can calculate

$$\begin{aligned}\frac{\partial \pi_i^T(x_i, K - x_i)}{\partial x_i} &= (r_i - w_i) - (r_i - s_i)(\Delta_1 + \Delta_2 + \Delta_5) \\ &\quad - (r_i - c_{ji})(\Delta_3 + \Delta_4) + (c_{ij} - t_{ij} - s_i)(\Delta_1 + \Delta_2)\end{aligned}$$

Using the fact that D_i and D_j are uniformly distributed in $[0, u]$ with $u \geq K$ to calculate the probabilities, and noting $c_{ij} - t_{ij} - s_i \leq \frac{r_i - s_i}{2} \leq c_{ji} - s_i$, we have

$$\begin{aligned}\frac{\partial^2 \pi_i^T(x_i, K - x_i)}{\partial x_i^2} &= -\frac{K - x_i}{u}(c_{ji} - s_i) \\ &\quad - \frac{u - (K - x_i)}{u}(r_i - (c_{ij} - t_{ij}))\frac{u - (K - x_i)}{u} \\ &\quad + \frac{u - x_i}{u}(r_i - c_{ji}) + \frac{x_i}{u}(c_{ij} - t_{ij} - s_i) \\ &\leq -\frac{u - K}{u}(c_{ji} - c_{ij} + t_{ij}) \\ &\leq 0.\end{aligned}$$

Therefore, $\pi_i^T(x_i, K - x_i)$ is quasi-concave. ■

Proof of Proposition 2 When $y_j = 0$, $\Pi_i^{TL}(y_i, 0) = \pi_i^T(y_i, 0)$. $\Pi_i^{TL}(y_i, 0)$ is concave in $y_i \geq 0$ (Rudi et al., 2001). Consider $0 < y_j \leq K$. Note that $\Pi_i^{TL}(y_i, y_j)$ is continuous in y_i and y_j . We show the quasi-concavity of $\Pi_i^{TL}(y_i, y_j)$ for $0 \leq y_i < K - y_j$, for $K - y_j < y_i \leq K$ and at $y_i = K - y_j$, respectively.

If $0 \leq y_i < K - y_j$, then $\mathbf{x} = \mathbf{y}$ and $\Pi_i^{TL}(y_i, y_j) = \pi_i^T(y_i, y_j)$. Rudi et al. (2001) proved that $\pi_i^T(y_i, y_j)$ is concave in y_i for any given $0 < y_j \leq K$.

If $K - y_j < y_i \leq K$, then $\Pi_i^{TL}(y_i, y_j) = \pi_i^T(\varphi_i(y_i, y_j), K - \varphi_i(y_i, y_j))$. Consequently,

$$\frac{\partial \Pi_i^{TL}(y_i, y_j)}{\partial y_i} = \frac{\partial \pi_i^T(\varphi_i(y_i, y_j), K - \varphi_i(y_i, y_j))}{\partial \varphi_i(y_i, y_j)} \frac{\partial \varphi_i}{\partial y_i}.$$

The increasing condition (III) of $\varphi_i(y_i, y_j)$ implies that $\frac{\partial \varphi_i}{\partial y_i} \geq 0$. Then $\Pi_i^{TL}(y_i, y_j)$ is quasi-concave for $K - y_j < y_i \leq K$ if and only if $\pi_i^T(x_i, K - x_i)$ is quasi-concave for $K - y_j < x_i \leq \varphi_i(K, y_j)$. Note that the union of intervals $K - y_j < x_i \leq \varphi_i(K, y_j)$ over $0 < y_j \leq K$ is $0 < x_i < K$. Hence, $\Pi_i^{TL}(y_i, y_j)$ is quasi-concave for $K - y_j < y_i \leq K$ with any $0 < y_j \leq K$ being given if and only if $\pi_i^T(x_i, x_j)$ is quasi-concave for $0 < x_i < K$.

At $y_i = K - y_j$, $\Pi_i^{TL}(y_i, y_j)$ may not be differentiable with respect to y_i . We examine the left and right partial derivatives of $\Pi_i^{TL}(y_i, y_j)$. If $R_i^T(x_j)$ has no intersection with the supply line $\{(x_1, x_2) | x_1 + x_2 = K, x_1, x_2 \geq 0\}$, then $\frac{\partial \Pi_i^{TL}(y_i, y_j)}{\partial y_i} \Big|_{(K-y_j)^-} \geq 0$ and $\Pi_i^{TL}(y_i, y_j)$

is quasi-concave at $y_i = K - y_j$. If $R_i^T(x_j)$ intersects the supply line at $(x_i^{I,K}, K - x_i^{I,K})$, there are two cases. If $K - y_j \leq x_i^{I,K}$, then $\frac{\partial \Pi_i^{TL}(K - y_j, y_j)}{\partial y_i} \geq 0$ and $\Pi_i^{TL}(y_i, y_j)$ is quasi-concave at $y_i = K - y_j$. Otherwise, $K - y_j > x_i^{I,K}$, then $\frac{\partial \Pi_i^{TL}(y_i, y_j)}{\partial y_i} \leq 0$. By the condition $x_i^{M,K} \leq R_i^T(K - x_i^{M,K})$, we know that $x_i^{M,K} \leq x_i^{I,K}$. Then $\frac{\partial \Pi_i^T(\varphi_i(K - y_j, y_j), K - \varphi_i(K - y_j, y_j))}{\partial \varphi_i(y_i, y_j)} \leq 0$ and $\frac{\partial \Pi_i^{TL}(y_i, y_j)}{\partial y_i} \leq 0$. In this case, $\Pi_i^{TL}(y_i, y_j)$ is quasi-concave at $y_i = K - y_j$.

Therefore, $\Pi_i^{TL}(y_i, y_j)$ is quasi-concave for $0 \leq y_i \leq K$ given any $0 \leq y_j \leq K$, $i, j = 1, 2$, $i \neq j$.

Now we prove the equivalency of $x_i^{M,K} \leq R_i^T(K - x_i^{M,K})$ and $\frac{r_i - c_{ji}}{c_{ij} - t_{ij} - s_i} \geq \frac{\Pr(D_i + D_j \leq K \leq x_i^{M,K} + D_j)}{\Pr(x_i^{M,K} + D_j \leq K \leq D_i + D_j)}$. $x_i^{M,K} \leq R_i^T(K - x_i^{M,K})$ is equivalent to $\frac{\partial \pi_i^T(x_i^M, x_i)}{\partial x_i} \geq 0$ which is written out as

$$(r_i - w_i) - (r_i - s_i)(\Delta_1 + \Delta_2 + \Delta_5) - (r_i - c_{ji})\Delta_3 + (c_{ij} - t_{ij} - s_i)\Delta_2 \geq 0.$$

We know that $\frac{\partial \pi_i^T(x_i^M, K - x_i^M)}{\partial x_i} = 0$ which is

$$\begin{aligned} & \frac{\partial \pi_i^T(x_i, K - x_i)}{\partial x_i} \\ &= (r_i - w_i) - (r_i - s_i)(\Delta_1 + \Delta_2 + \Delta_5) \\ & - (r_i - c_{ji})(\Delta_3 + \Delta_4) + (c_{ij} - t_{ij} - s_i)(\Delta_1 + \Delta_2) = 0. \end{aligned}$$

Therefore,

$$(r_i - c_{ji})\Delta_4 - (c_{ij} - t_{ij} - s_i)\Delta_1 \geq 0$$

which is equivalent to $\frac{r_i - c_{ji}}{c_{ij} - t_{ij} - s_i} \geq \frac{\Pr(D_i + D_j \leq K \leq x_i^{M,K} + D_j)}{\Pr(x_i^{M,K} + D_j \leq K \leq D_i + D_j)}$. ■

Proof of Proposition 3 By calculation,

$$\frac{\partial \pi_i^T(K, 0)}{x_i} = (r_i - w_i) - (c_{ij} - t_{ij} - s_i)\Delta_1 - (r_i - c_{ij} + t_{ij})(\Delta_1 + \Delta_2)$$

When K is sufficiently small, we know Δ_1 and Δ_2 are small which makes $\frac{\partial \pi_i^T(K, 0)}{x_i} > 0$. When K is sufficiently large, we know $\Delta_1 = 1$ and $\Delta_2 = 0$ which makes $\frac{\partial \pi_i^T(K, 0)}{x_i} = -w_i + s_i < 0$. Note that both Δ_1 and $\Delta_1 + \Delta_2$ are non-decreasing in K which means $\frac{\partial \pi_i^T(K, 0)}{x_i}$ is non-increasing in K . Therefore, there exists K_i^0 such that $\frac{\partial \pi_i^T(K, 0)}{x_i} \geq 0$ for all $K \leq K_i^0$ and $\frac{\partial \pi_i^T(K, 0)}{x_i} \leq 0$ for all $K > K_i^0$. By the quasi-concavity of $\pi_i^T(x_i, K - x_i)$, we know $x_i^{M,K} = K$ when $K \leq K_i^0$.

When $K > K_i^0$, $x_i^{M,K}$ satisfies $\frac{\partial \pi_i^T(x_i^{M,K}, K - x_i^{M,K})}{\partial x_i} = 0$. Hence,

$$\frac{dx_i^{M,K}}{dK} = -\frac{\partial^2 \pi_i^T(x_i^{M,K}, K - x_i^{M,K}) / \partial K \partial x_i}{\partial^2 \pi_i^T(x_i^{M,K}, K - x_i^{M,K}) / \partial x_i^2}.$$

By calculation, we have $\frac{\partial^2 \pi_i^T(x_i^{M,K}, K - x_i^{M,K})}{\partial K \partial x_i} = -(r_i - c_{ji})\delta_4 - (c_{ij} - t_{ij} - s_i)\delta_3 \leq 0$. By the quasi-concavity of $\pi_i^T(x_i, K - x_i)$, we have $\frac{\partial^2 \pi_i^T(x_i^{M,K}, K - x_i^{M,K})}{\partial x_i^2} < 0$. Therefore, $\frac{dx_i^{M,K}}{dK} \leq 0$ which implies that $x_i^{M,K}$ is non-increasing in K .

Consider a continuous function $\rho(K) = K - (x_1^{M,K} + x_2^{M,K})$. Note that $\rho(K) < 0$ when $0 < K < \max\{K_1^0, K_2^0\}$. On the other hand, $\rho(K) > 0$ when K is sufficiently large since $x_1^{M,K}$ and $x_2^{M,K}$ are non-increasing in K . Therefore, there must be $\underline{K}$ such that $\rho(\underline{K}) = 0$, which is equivalent to $\underline{K} = (x_1^{M,\underline{K}} + x_2^{M,\underline{K}})$.

At $\bar{K}$, we have $x_i^{M,\bar{K}} \leq x_i^{N,T}$, $i = 1, 2$. Then $\rho(\bar{K}) = 8\bar{K} - (x_1^{M,\bar{K}} + x_2^{M,\bar{K}}) = (x_1^{N,T} + x_2^{N,T}) - (x_1^{M,\bar{K}} + x_2^{M,\bar{K}}) \geq 0$. Therefore, $\underline{K} \leq \bar{K}$. ■

Proof of Theorem 1 The theorem follows Table 2 and the preceding analysis. ■

Proof of Theorem 2 For case (i), we have $\pi_i^T(x_i^{N,T}, x_j^{N,T}) \geq \pi_i^T(x_i^{N,L}, x_j^{N,T}) \geq \pi_i(x_i^{N,L})$. For case (ii), we have $\pi_i^T(x_i^{M,K}, K - x_i^{M,K}) \geq \pi_i^T(x_i, K - x_i)$ for all $0 \leq x_i \leq K$, and $\pi_i^T(x_i^{M,K}, K - x_i^{M,K}) \geq \pi_i^T(x_i^*, K - x_i^*) \geq \pi_i(x_i^*)$. For case (iii), we know $x_i^{M,K} \geq x_i^{N,TL} \geq x_i^{N,L}$. Then $\pi_i^T(x_i^{N,TL}, K - x_i^{N,TL}) \geq \pi_i^T(x_i^{N,L}, K - x_i^{N,L}) \geq \pi_i(x_i^{N,L})$ by the quasi-concavity of $\pi_i^T(x_i, K - x_i)$. ■

Proof of Propositions 4 and 5 The two propositions follow Theorem 2 and Table 4. ■

Proof of Proposition 6 Reorganize Equation 6 and note that $\alpha_i = 1 - \alpha_j$, $i, j = 1, 2$, $i \neq j$. ■

Proof of Theorem 3 Note $\frac{\partial \Pi_i^{TL}(y_i, y_j; \mathbf{c}(y))}{\partial y_i} = \alpha_i \frac{\partial \Pi_i^{TL}(y)}{\partial y_i}$ and the theorem follows. ■

APPENDIX B: ORDER LIMITS

In Models NTL and NL, we assume both newsvendors' order quantities be restricted by the same order limit K , which is the supplier's capacity. We point out that lifting the order limit restriction may make both newsvendors better off in some cases, but not all. Revisit Example 2. Assume newsvendor i 's order limit to be K_i , $i = 1, 2$. For a potential

equilibrium inventory allocation $\mathbf{x}^{N,TL}$, it is easy to calculate that $\Pi_1^{TL}(x_1^{N,TL}, K - x_1^{N,TL}) > \Pi_1^L(y_1^{N,L}, y_2^{N,L})$ and $\Pi_2^{TL}(K - x_1^{N,TL}, x_1^{N,TL}) > \Pi_2^L(y_2^{N,L}, y_1^{N,L})$ when $5.06 < x_1^{N,TL} \leq 6.00$. Hence, both newsvendors will be better off if $\frac{5.06}{K-5.06} < \frac{K_1}{K_2} \leq \frac{6.00}{K-6.00}$ which is equivalent to $1.90K_2 \leq K_1 \leq 3.00K_2$. Only if the order limits satisfy these conditions can both newsvendors be better off.

In a general setting with newsvendors' order limits being K_1 and K_2 , Theorem 2, Propositions 4 and 5 still hold with $\frac{K_i}{2}$ replaced by $\frac{K_i}{K_1+K_2}K$ in case (iii) of Proposition 5. Therefore, simply setting arbitrary order limits or a large order limit for one newsvendor does not necessarily ensure that both newsvendors are better off. Actually, if there exists no inventory allocation $(x_1, K - x_1)$, $K - x_2^{M,K} \leq x_1 \leq x_1^{M,K}$ which can make both newsvendors better off, then it is impossible for the inventory transshipment to benefit both newsvendors by solely relaxing ordering limits.

APPENDIX C: QUASI-CONCAVITY OF $\pi_i^T(x_i, K - x_i)$

In this paper we assume that the profit function $\pi_i^T(x_i, K - x_i)$ is quasi-concave for $i = 1, 2$. Our numerical study shows that this is a weak condition despite that theoretically, it is possible that $\pi_i^T(x_i, K - x_i)$ could be not quasi-concave. In one of the experiments, we randomly generate the parameters within certain ranges. Specifically, we set $r_1 = r_2 \sim U[0, 30]$, $w_1 = w_2 \sim U[0, r_1]$, $s_1 = s_2 \sim U[0, w_1]$, $c_{12} \sim U[s_1, r_1]$, $c_{21} \sim U[s_2, r_2]$, $t_{12} \sim U[0, c_{12} - s_1]$, $t_{21} \sim U[0, c_{21} - s_2]$, $K \sim U[5, 50]$ and K is an integer. We let demand D_1 and D_2 be independent. We have tested uniform, normal and lognormal distributions as the demands. For uniform distribution, D_i is uniformly distributed in $[l_i, u_i]$ with $l_i \sim U\left[0, \frac{K}{2}\right]$ and $u_i \sim U\left[\frac{K}{2}, \frac{3K}{2}\right]$, $i = 1, 2$. For normal distribution, D_i is normally distributed with mean μ_i and standard deviation σ_i with $\mu_i \sim U\left[\frac{K}{4}, \frac{3K}{2}\right]$ and $\sigma_i \sim U\left[0, \frac{\mu_i}{3}\right]$, $i = 1, 2$. For lognormal distribution, D_i is lognormally distributed with μ_i being mean of $\ln(D_i)$ and σ_i being standard deviation of $\ln(D_i)$ and with $\mu_i \sim U\left[0, \ln\left(\frac{3K}{2}\right)\right]$ and $\sigma_i \sim U[0, \ln(2K)]$, $i = 1, 2$. For each distribution family, we have examined 100 000 randomly generated cases and have observed that $\pi_1^T(x_1, K - x_1)$ and $\pi_2^T(x_2, K - x_2)$ are quasi-concave in all cases.

APPENDIX D: ENDOGENOUS WHOLESALE PRICE

We consider a variant of Model NTL in which the wholesale prices can be endogenously decided by the supplier before the newsvendors negotiate the transshipment prices by the coordinating mechanism in Section 6. Let w_0 denote the production cost of the supplier. For simplicity, we assume

that the supplier offers the same wholesale price to both newsvendors, that is, $w_1 = w_2 = w \geq w_0$. For any order $\mathbf{y}$ and its inventory allocation $\mathbf{x} = (\varphi_1(y_1, y_2), \varphi_2(y_2, y_1))$, we use $\pi_0(\mathbf{x}|w) = (w - w_0)(x_1 + x_2)$, $\pi_i^T(\mathbf{x}|w) = \pi_1^T(\mathbf{x}|w) + \pi_2^T(\mathbf{x}|w)$ and $\pi_c(\mathbf{x}) = \pi_0(\mathbf{x}|w) + \pi_i^T(\mathbf{x}|w)$ to denote the supplier's profit, newsvendors' total profit and the supply chain profit, respectively. From Section 6 we know that for any given K and w , the newsvendors will order the Nash equilibrium such that the inventory allocation maximizes $\pi_i^T(\mathbf{x}|w)$. Denote this inventory allocation as $\mathbf{x}^R(w) = (x_1^R(w), x_2^R(w))$ and denote the inventory allocation that maximizes $\pi_c(\mathbf{x})$ as $\mathbf{x}^c$.

When $w = w_0$, supplier has zero profit because $\pi_0(\mathbf{x}|w_0) = 0$. Therefore, $\mathbf{x}^R(w_0) = \mathbf{x}^c$ which means the newsvendors' orders maximize the supply chain profit. Suppose the supplier increases w to be higher than w_0 . We consider two possibilities. First, if K is large enough to satisfy $x_1^R(w_0) + x_2^R(w_0) < K$, then it implies that $\frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} = \frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} = 0$. Note that $\frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w)}{\partial x_i} = \frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} - (w - w_0) < 0$, $i = 1, 2$. Therefore, $\mathbf{x}^R(w) \neq \mathbf{x}^R(w_0)$ and equivalently, $\mathbf{x}^R(w) \neq \mathbf{x}^c$. This implies that, if the supplier wants a positive profit, the supply chain profit will not be optimized. Second, if K is relatively small so that $x_1^R(w_0) + x_2^R(w_0) = K$, then by the KKT conditions of the optimization problem

$$\begin{aligned} \max \quad & \pi_i^T(\mathbf{x}|w_0) \\ \text{s.t.} \quad & x_1 + x_2 \leq K \\ & x_1, x_2 \geq 0, \end{aligned}$$

we have $\frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} = \frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} \geq 0$. Without loss of generality we assume $\frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} = \frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_2} > 0$. Hence, when $w_0 \leq w \leq \frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} + w_0$, we have $\frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w)}{\partial x_i} = \frac{\partial \pi_i^T(\mathbf{x}^R(w_0)|w_0)}{\partial x_i} - (w - w_0) \geq 0$, $i = 1, 2$, and it remains that $\mathbf{x}^R(w) = \mathbf{x}^c$. This implies that, when w is within a certain range, the supplier can extract a positive profit while the supply chain profit is maximized.

APPENDIX E: MULTIPLE NASH EQUILIBRIA WHEN $\underline{K} < K < \bar{K}$

When $\underline{K} < K < \bar{K}$, there will be infinitely many Nash equilibria. We point out that all of them are Pareto optimal. Actually, for any Nash equilibrium $(y_1^{N,TL}, y_2^{N,TL})$ and its inventory allocation $(x_1^{N,TL}, x_2^{N,TL})$, we know that $x_1^{N,TL} \geq x_1^{M,K}$ and $x_2^{N,TL} \geq x_2^{M,K}$. Consider the inventory allocation $(\tilde{x}_1^{N,TL}, \tilde{x}_2^{N,TL})$ of another Nash equilibrium. Without loss of generality we may assume $x_1^{N,TL} > \tilde{x}_1^{N,TL}$ and therefore, $x_2^{N,TL} < \tilde{x}_2^{N,TL}$. By the quasi-concavity of $\pi_1^T(x_1, K - x_1)$ and $\pi_2^T(x_2, K - x_2)$, we have $\pi_1^T(x_1^{N,TL}, x_2^{N,TL}) < \pi_1^T(\tilde{x}_1^{N,TL}, \tilde{x}_2^{N,TL})$ and $\pi_2^T(x_2^{N,TL}, x_1^{N,TL}) > \pi_2^T(\tilde{x}_2^{N,TL}, \tilde{x}_1^{N,TL})$, which means that $(x_1^{N,TL}, x_2^{N,TL})$ is a Pareto

optimal equilibrium inventory allocation, and therefore, its corresponding Nash equilibrium $(y_1^{N,TL}, y_2^{N,TL})$ is a Pareto optimal Nash equilibrium.

Moreover, if in Model NL all supply is allocated, that is, $x_1^{N,L} + x_2^{N,L} = K$, then Proposition 4 still holds that at least one newsvendor is better off in Model NTL. To see this, note that we may assume $x_1^{N,TL} < x_1^{N,L}$ and $x_2^{N,TL} > x_2^{N,L}$. Then $\pi_1^T(x_1^{N,TL}, x_2^{N,TL}) \geq \pi_1^T(x_1^{N,L}, x_2^{N,L}) \geq \pi_1(x_1^{N,L})$. In this case newsvendor 1 is better off in Model NTL than in Model NL.

APPENDIX F: MULTIPLE NEWSVENDORS

The modeling platform and analysis in this paper can be extended to the general case of more than two newsvendors. Different from the two-newsvendor case, a newsvendor may transship inventory to more than one newsvendor with excess demand, each of which may purchase from multiple newsvendors with surplus. Consequently, an inventory

transshipment mechanism needs to be specifically defined. Such a mechanism dictates the allocation of the transshipment profit among the newsvendors. Existing research generally takes a centralized approach for determining the transshipment arrangement and directly focuses on allocation methods of the transshipment profit. For example, Anupindi et al. (2001) formulate the transshipment problem as a linear programming problem and use the dual variables as transshipment prices. Huang and Sošić (2010b) compare this dual allocation method with the method based on pre-fixed transshipment prices and propose to introduce a central depot to simplify the transshipment. Hanany et al. (2010) design a transshipment fund mechanism to better manage the transshipment and the subsequent profit allocation. Fang and Cho (2014) focus on network structures and the system stability under different allocation mechanisms. The insights revealed in this paper may help understand the effect of transshipment prices and mechanisms in supply chains with multiple newsvendors.