

A Mixed-Integer Linear Programming Method for A Two-dimensional Material Slitting Problem

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Abstract

This paper is based on a real-life application where big rolls (master rolls) of laminated films are slitted into smaller sizes as per customer specifications. The master rolls are of a constant length and varying widths. The dimensions of the smaller rolls vary based on the customer orders. The aim is to achieve less scrap while slitting the master rolls to meet customer demand. We propose a mixed-integer linear programming approach for the problem. A column generation-based method is developed and implemented to reduce the total scrap during the slitting process. An example problem instance is solved by using the proposed approach to compare with the result using the existing manual method. The findings of this project will directly benefit the companies, similar to the one studied, in minimizing scrap.

Keywords: Slitting, Cutting stock problem, Column generation, Laminates, Scrap reduction

1. Introduction

A thermal lamination film is a decorative film that is extruded using a thermal adhesive. Laminated films are made using resin or a thermal adhesive that can melt even at low temperatures. These film rolls vary based on the type of finish needed. These films are used in the printing, publishing, and packaging industries. This project was done in a company where master rolls of laminated films are outsourced and cut into required sizes based on customer orders. A slitting machine is used to slit the master rolls. The master rolls are loaded to one side of the machine and the finished product (smaller rolls) comes out on the other side. The slitting machine has blades and through programming the dimensions to cut are fed into the machine. This paper discusses how to improve the slitting process of master rolls of thermal film into smaller sizes. Improper planning leads to large amount of scrap. Scrap is any left-over or unusable material formed as a byproduct of the slitting process. Moreover, materials that do not meet the design, quality, or customer requirements are also considered scrap. There are a lot of issues with recycling the thermal laminated films, as it is difficult to separate the resin from the substrate. Scrap not only causes financial distress to the company, but it also impacts our environment as it cannot be recycled.

The current system relies on a scheduler who manually plans the cutting pattern/sequence and schedules orders on the slitting machine. The current scheduling system is not effective and time consuming. The manual method also incurs a huge amount of scrap. Moreover, the lead time is longer due to delays and manual planning. The proposed method can reduce scrap and lower the planning time from weeks to minutes, which leads to increased productivity with a shorter lead time. Due to the non-disclosure agreement with the company, the name of the company and other proprietary information cannot be revealed. However, in general, the problem under study can be noticed in industries such as paper mills, sheet metal fabrications, garment industry, etc.

2. Problem Description

The master rolls of thermal films come in a standard length and a few different widths. The dimensions of the smaller rolls vary depending on the customer. The customer orders are batched based on the unique length of the order because the slitting machine can process orders of the same length. The master roll is manually loaded to the machine on one side. The operator then has to set a series of blades to cut varying widths of smaller rolls. However, the machine can only process different customer orders as a batch if the length of the roll demanded is the same. So, the scheduler tries to consolidate customer orders of same length with varying widths to minimize the scrap.

The problem discussed here is similar to the Cutting Stock Problem (CSP). In CSP a master roll is cut into smaller pieces of known widths of constant length. However, for the problem under study the customer orders are of varying

lengths and widths. The CSP has many applications in the field of wood, textiles, leather, and other industries. The objective is to minimize the scrap and number of master rolls used. **Figure 1** illustrates the problem. The problem under consideration is similar to a two-dimensional CSP. However, the master rolls come in different widths.

In general, CSP does not consider the master roll of varying widths and smaller rolls of changing lengths. To overcome the shortcoming of CSP, a Mixed Integer Linear Program (MILP) was used to solve the Two-Dimensional Cutting Stock Problem (2DCSP). 2DCSP, where infinitely many master rolls of length L , width W is given and a list of n rolls of length l_i and width w_i is cut d_i times. The 2DCSP requires cutting all the items by minimizing the number of master rolls used [1]. However, the formulation proposed in this study aims to minimize the number of master rolls used and the scrap. The column generation method has been used to solve the problem. The dimensions of the standard master rolls and their quantity in stock are known. Similarly, the length and width of the smaller rolls and the demand are also known from the customer. The commercial solver IBM ILOG CPLEX is used to solve the proposed MILP.

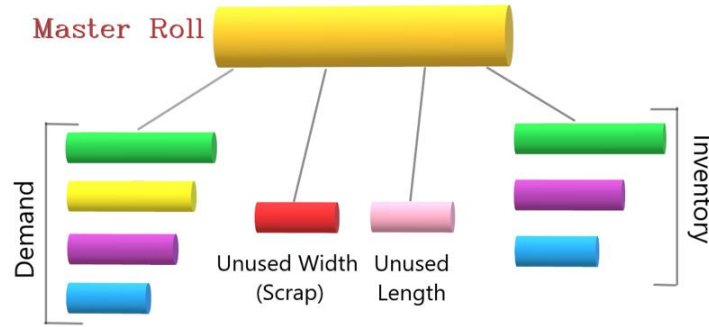


Figure 1. Slitting a master roll

3. Related Research

The literature shows that 2DCSP can be classified into several categories based on the constraints. Certainly, the cutting pattern can be guillotine or non-guillotine cuts, oriented or non-oriented. In guillotine cutting the position or the orientation of the cutting tool cannot be changed. Ayadi et al. [2] proposed a new formulation for two-dimensional guillotine non-oriented CSP combining firm and forecasted orders. Ayadi et al. [2] used a hybrid heuristic based on the bottom left and shelf algorithms to solve the problem. They concluded that the hybrid heuristics gave better results in terms of waste and the number of sheets used. Israni and Sanders [3] reviewed 2DCSP and explained the difference between 1-D and 2-D CSP. But the 2-D problem was guillotine cutting patterns. Israni and Sanders [3] proposed an algorithm based on first fit heuristics for the 2-D Guillotine cutting pattern. It was shown that trim losses are less than most of the algorithms.

Silva et al. [4] proposed pseudo-polynomial size models two and three-stage dimensional 2DCSP guillotine patterns. The problem involves cutting small-sized rectangles from a larger rectangle of standard size. Gilmore and Gomory [5] proposed a linear programming formulation for the two-dimensional cutting stock problem involving guillotine cutting pattern. Furini and Malaguti [6] proposed a formulation for 2DCSP with guillotine cuts comparing three different MILPs. Vanderbeck [7] used the nested column generation (CG) approach in an approximate fashion to a two-dimensional cutting stock problem. The cutting pattern involves three stages of orthogonal guillotine cutting. The main objective is to minimize waste. Previous literature did not include the machine constraint that we have (i.e., rolls of the same length should be processed together). Most of the research was concentrated on guillotine cutting patterns. There was no exact algorithm to solve 2DCSP.

4. Methodology

The problem under study can be reduced to a one-dimensional cutting stock problem which is *NP-hard* and usually solved by the column generation (CG) approach. We formulate the problem and design the solution method by the CG framework. CG technique has been implemented to solve larger problem instances which exhibit a block diagonal structure. According to Rardin [8], "Column generation approaches deal with complex combinatorial problems by first enumerating a sequence of columns representing viable solutions to parts of the problem, and then solving a set

partitioning (or covering or packing) model to select an optimal collection of these alternatives fulfilling all problem requirements.” In the Column Generation (CG) approach, the original formulation is decomposed into a restricted master problem and one or more subproblem(s). The restricted master problem is then linearly relaxed and solved iteratively to optimality with the improving columns generated by solving the subproblem. Finally, the restricted master problem is resolved as an integer program to obtain a final feasible solution [9]. The slitting machines had limitations that should be taken into consideration. A mathematical formulation was developed considering all the constraints including the machine, raw materials, and customer demand.

In the traditional CSP, the length of the roll remains constant. But in this project, the lengths and widths of the smaller rolls vary. Moreover, customer demands for rolls with the same length should be processed together. So, scheduling and batching of orders should be based on the length of the rolls. In this paper, CG is used to generate the cutting patterns. These cutting patterns are generated as improving columns by solving the subproblem. As mentioned earlier, only orders of the same length can be processed in the machine; the subproblem is called for the unique length of orders and solved for different widths of the master roll available. This step is repeated until there is no improving column. The column generation steps are:

1. Solve the restricted master problem using the initial feasible solution
2. Obtain the dual values
3. Identify number of unique lengths of smaller rolls
4. Pass the dual values to the sub problem
5. Solve the sub-problem based on unique length for different width of the master roll
 - a. Find the column with the most negative reduced cost.
6. Add the column(s) to the set of columns in the restricted master problem.
7. Repeat the steps until there is no improving column
8. Use the generated columns to solve the restricted integer problem.

The mixed integer linear programming model for the proposed problem is discussed in this section. A summary of the notation used is presented below.

Indices

I	Set of unique lengths of orders
$J(i)$	Set of items j in unique length group $i \in I$
K	Set of master roll widths
$M(k)$	Set of master rolls in width group $k \in K$
$P(i)(k)$	Set of cutting patterns for orders in unique length group i for master rolls width group k

Parameters

l_i	Length of customer orders in unique length group i
w_{ij}	Width of item j in order group i
d_{ij}	Demand for item j in order group i
r_{pj}	Number of item j in pattern $p \in P(i)(k)$ for $i \in I, k \in K$
n_{ij}	Quantity of item j in order group i to be stocked in inventory
L	Length of master roll
W_m	Width of master roll $m \in M(k)$ for $k \in K$
α	Weight used in objective function; details below
Q_k	Quantity of master roll on-hand in width group k
$BigM$	A large positive number

Decision Variables

y_{pm}	Number of times pattern p is cut from the master roll m
θ_m	Unused length of master roll m
δ_{ij}	Number of additional units of item j in order group i to be stocked as excess inventory
a_m	Binary variable, 1 if master roll m is used; 0 otherwise

The restricted master problem for the column generation approach is as follows.

Minimize

$$\sum_{k \in K} \sum_{m \in M(k)} \sum_{i \in I} \sum_{p \in P(i)(k)} l_i \left(W_m - \sum_{j \in J(i)} r_{pj} w_{ij} \right) y_{pm} + \alpha \sum_{k \in K} \sum_{m \in M(k)} L W_m a_m + \sum_{k \in K} \sum_{m \in M(k)} a_m \quad (1)$$

Subject to:

$$\sum_{i \in I} \sum_{p \in P(i)(k)} l_i y_{pm} + \theta_m = L a_m \quad \forall k \in K, m \in M(k) \quad (2)$$

$$\sum_{k \in K} \sum_{m \in M(k)} \sum_{p \in P(i)(k)} r_{pj} \times y_{pm} - \delta_{ij} = d_{ij} \quad \forall i \in I, j \in J(i) \quad (3)$$

$$\sum_{m \in M(k)} a_m \leq Q_k \quad \forall k \in K \quad (4)$$

$$\sum_{p \in P(i)(k)} y_{pm} \leq \text{BigM} \times a_m \quad \forall k \in K, m \in M(k) \quad (5)$$

$$\delta_{ij} \leq n_{ij} \quad \forall j \in J(i) \quad (6)$$

$$\theta_m \leq L \times a_m \quad \forall k \in K, m \in M(k) \quad (7)$$

$$y_{pm}, \theta_m, \delta_{ij} \geq 0, a_m = 0 \text{ or } 1, y_{pm} \text{ integer} \quad (8)$$

The objective function (1) minimizes the width scrap, total area used to cut in the master roll and number of master rolls used. The total area used is multiplied by a value α which is the user defined value and can be changed before solving the problem. The three possible values of α used in practice are given in **Table 1** to indicate the preference of creating less unused length of master rolls. The third term in (1) denotes the number of master rolls used regardless of the α value. Constraint (2) is the length constraint which ensure the items cut does not exceed the master roll length (L). Constraint (3) is the demand constraint where total items cut is equal to demand. Constraint (4) ensures that total master roll used is less than or equal to the available quantity. Constraint (5) means master roll m must be used if any pattern is cut from it. Constraint (6) ensures that extra item δ_{ij} cut is less than or equal to the allowed inventory n_{ij} . This shows inventory requirement of an item may or may not be satisfied in order to minimize scrap. Constraint (7) ensures that the unused length θ_m is less than master roll length L . Constraint (8) is the non-negativity constraint. When solving the restricted master problem, the binary constraint and integer constraint is relaxed.

Table 1. α values

α	Meaning
0	It is acceptable to use more master rolls to achieve a lower scrap.
0.5	Balance scrap and master rolls used.
1	Use less master rolls. But scrap might be higher.

A sub problem is solved for each unique width of a master roll and all customer orders with same length.

Minimize

$$L_i \left(W_m - \sum_{j \in J(i)} r_j w_j \right) - (\max(m \in M(k))(u_m \times L_i)) - \sum_{j \in J(i)} v_j \times r_j - (\max(m \in M(k))t_m) \quad (9)$$

Subject to:

$$\sum_{j \in J(i)} w_j r_j \leq W_m \quad (10)$$

$$r_j \leq d_j + n_j \quad (11)$$

$$r_j \geq 0 \quad (12)$$

The objective function of the sub problem (9) minimizes the dual values to use the master roll width as much as possible. u, v, t are the dual variable values of constraints (2), (3), and (5). Constraint (10) ensures that the width of all items in a pattern does not exceed the master roll width. Items present should be less than the sum of demand and inventory is ensured by constraint (11). Constraint (12) is the non-negativity constraint.

5. Results

An example problem is solved both manually (following the current practice at the company where this project was conducted) and with the formulation proposed. The data in **Table 2** mimics real time data typically observed. Due to the non-disclosure agreement, real data cannot be used. The unused length of a master roll is not considered as scrap in our model as this might be reused for future orders. A production planner at the company decides manually how to slit the master roll to meet the customer demand purely based on their past experience. As mentioned earlier this is a time-consuming process and results in huge scrap. **Figure 2** shows the cutting pattern decided by the production planner in the company. **Figure 3** shows the results based on proposed formulation solved using commercial solver IBM ILOG CPLEX.

Table 2. Example Data

Length (ft)	Width (inches)	Demand (units)	Inventory (units)	Master roll Width (inches)	Mater roll Quantity (units)
5000	9.5	4	0	30	2
5000	20	2	5	35	4
5000	13	5	2	40	4
5000	19	5	4	45	3
5000	11	7	0		
6000	13	6	0	Master roll Length	40000 ft
6000	10	4	0	α	1
6000	12	4	0		
10000	13	2	0		
10000	10	5	0		
10000	14.5	4	0		

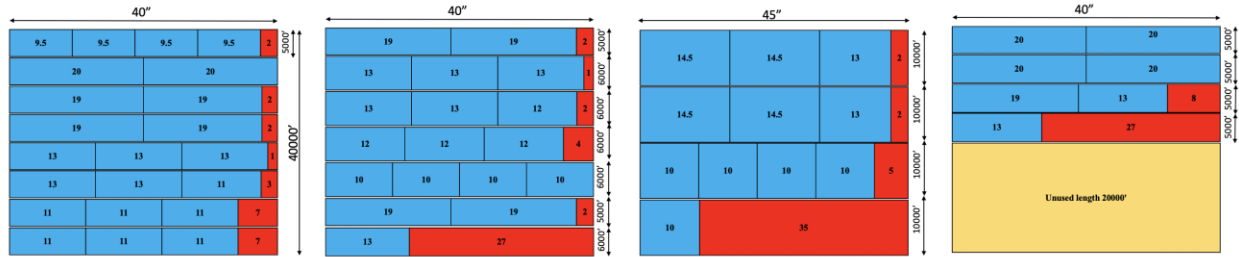


Figure 2. Manual results

Scrap percentage is calculated as $\frac{\text{Total area scraped}}{\text{Total area used to cut}} \times 100\%$ for both methods. This is the formula used by the company. **Table 3** is the comparison of results. From the results, the scrap percentage and time consumed to define the cutting patterns has reduced drastically. This is because the production planner cannot explore all the feasible cutting patterns available for different master roll width. The CG approach explores more feasible cutting patterns as a result a better solution is obtained. As the number of customer orders increases the proposed formulation will have lower scrap percentage and less time to solve the problem. A user interface with MS Excel was created in the final product for the company. With this interface the order details are entered, and the program is run using Excel macros and results are exported in MS Excel.

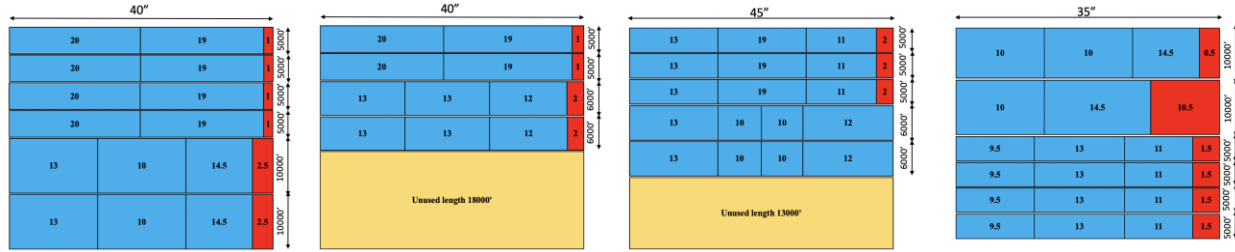


Figure 3. CG results - based on proposed formulation

Table 3. Comparison table

	Manual Method	CG – Proposed formulation
Scrap %	16.53%	5.37%
Time consumed	35 min	3 min
Cutting patterns generated	15	966

6. Conclusions

The findings of this project directly benefit the schedulers in the company who are faced with the ardent task of scheduling hundreds of orders each day on their slitting machine. After several trial runs of the proposed system, the company was able to reduce their scrap percentage by over 50%, which results in huge savings. The scrap from the master rolls cannot be recycled and therefore, reducing the scrap reduces the amount of plastic waste dumped into the environment. The MILP proposed and the solution can benefit other industries which need to trim their raw material and minimize scrap (like lumber, paper mills, etc.). Initially, the company had a lead time of two months to plan and carry out the slitting operation. After the completion of this project, they were able to reduce their lead time to less than a month. In 2019, the total cost of scrap was almost half a million. This project resulted in a savings of over \$50,000. This work also increased the production efficiency of the company. A user interface was created so that anyone in the company can enter the details and run the program through MS Excel. In summary, the project benefits the company and helps to achieve their goal to reduce scrap.

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